# Revisiting preemptive overcontrol: Conditions and misconceptions in task completion behavior*

**Jong Jae Lee** [i]

*The Catholic University of Korea, Bucheon, Republic of Korea*

## Abstract

This paper investigates preemptive overcontrol exercised by a present-biased agent who is fully aware of her self-control problem. A usual understanding is that such an agent, a so-called sophisticate, performs a given task earlier than a time-consistent agent because she anticipates that even a single delay will cause her to complete the task beyond the ideal date. We first present a counterexample and provide a correct intuition by identifying the necessary conditions for preemptive overcontrol. We argue that a sophisticate would complete a task early when she anticipates that if she postpones, the net reward from completing the task will significantly decrease by the next task-completion date. Moreover, we show that if the net reward schedule is single-peaked, this intuition implies the usual understanding.

*Keywords*: Present-bias, Sophistication, Preemptive overcontrol, Quasi-hyperbolic discounting

JEL Classification: D11, D15, D91

---

* This paper has benefited from the careful reading and constructive comments of the editor and anonymous referees.

i) Department of Economics, The Catholic University of Korea, 43 Jibong-ro, Bucheon, Gyunggi 14662, Republic of Korea. Email: piuslee83@gmail.com

First received, September 8, 2024; Accepted, September 27, 2024

# 1 Introduction

People procrastinate tasks that incur immediate costs but yield delayed rewards. An extensive literature suggests that it stems from present bias: people's relative preferences for utilities at an earlier date over a later date becomes stronger as the earlier date gets imminent (Frederick et al., 2002; Augenblick et al., 2015; Augenblick and Rabin, 2019; Bisin and Hyndman, 2020). This present bias leads to the tendency to grab rewards immediately and to delay costs. However, agent's knowledge of her own present bias can counter the effect of present bias. O'Donoghue and Rabin (1999) study how agent's sophistication changes the behaviors of those who suffer from present bias problem. Specifically, in a stylized model of task-completion that an agent decides when to complete a single task within a given deadline, they show that a present-biased agent would complete a given task earlier if she is aware of her self-control problem. This so-called sophistication effect works in an opposite direction to the present bias effect and therefore mitigates it. This raises a question as to whether the sophistication effect dominates the present bias effect. O'Donoghue and Rabin (1999) do not answer but provide one example of "preemptive overcontrol" that a sophisticate preemptively performs a given task earlier than a time-consistent agent. A usual understanding is that a sophisticate performs at an earlier date because she anticipates even a single delay could result in procrastination, causing her to complete the task later than a time-consistent individual would.[1] To the best of our knowledge, conditions for preemptive overcontrol and the usual understanding about it have not been studied since O'Donoghue and Rabin (1999). In this paper, we first show that the usual understanding about preemptive overcontrol does not hold generally. In particular, we provide an example in which a single delay of a sophisticate on the task completion date does not necessarily cause her to complete a task beyond the ideal date (Example 2). In our example, even after a single delay, a sophisticate would complete a task earlier than a time-consistent individual would do. Then, we investigate as to what drives a

[1] Example 3 and the story behind it in O'Donoghue and Rabin (1999) are consistent with this intuition. Moreover, Ericson and Laibson (2019) state that preemptive overcontrol occurs "because they recognize if they delay today they will delay even more in the future."

sophisticate to preemptively overreact to her present bias. To this end, we derive implications of preemptive overcontrol (by identifying the necessary condition for it). Specifically, we find that the following conditions are required for preemptive overcontrol (Theorem 1): (i) neither rewards nor costs are constant across periods, (ii) the net rewards (rewards net of costs) do not have its maximum in the initial and terminal periods, (iii) a sophisticate would not perform a task on the ideal date when the net reward is maximized.

The necessary conditions for preemptive control imply that a sophisticate would complete a task early when she anticipates that if she postpones, the net reward from completing the task will significantly decrease by the next task-completion date (Proposition 1). The ideal date on which a time-consistent agent would perform a task cannot be the next task- completion date by (iii) and therefore it can be either earlier or later than the next task-completion date. We show that if the net reward is single-peaked around the optimal date, then a single delay before the ideal task completion date will push the task completion beyond that date for preemptive overcontrol to occur (Proposition 2). In other words, the usual explanation that a fear of a single delay leading to procrastination is necessary for preemptive overcontrol is correct when the net rewards are monotonically increasing by the optimal date and then decrease sharply afterwards. The three-date example of preemptive overcontrol in O'Donoghue and Rabin (1999) (replicated as Example 1 in this article) exhibits the same feature: the net reward shrinks from 9 on the task completion date (date 1) to 5 (on the next completion date (date 3) although the net reward the ideal date (date 2) is 10. The net reward is single-peaked and decreases sharper beyond the ideal date. Our result reveals that the usual understanding about preemptive overcontrol implicitly presumes the single-peakedness of the net reward schedule.

The rest of this article is organized as follows: Section 2 describes the model and the example of preemptive overcontrol in O'Donoghue and Rabin (1999), and Section 3 presents our main results. In Section 4, we conclude.

# 2 Preliminaries

## 2.1 Model

We consider the same environment studied in O'Donoghue and Rabin (1999) (henceforth, ODR99). For the sake of a self-contained presentation, we describe it here briefly.

An agent needs to complete a given task within $T$ periods, where $T$ is a positive integer. The task takes only one period to complete and therefore the agent decides whether to perform the task at each period $t = 1, 2, \ldots, T - 1$. If performing the task, the agent incurs cost $c_t$ but earns reward $v_t$. Let $\mathbf{v} \equiv (v_1, v_2, \ldots, v_T) \in \mathrm{R}^T_+$ and $\mathbf{c} \equiv (c_1, c_2, \ldots, c_T) \in \mathrm{R}^T_+$ be the reward and the cost schedules. The cost and the reward of performing the task in a period can be delivered at different dates within the same period. If the cost is incurred immediately (at the beginning of the period) and the reward is earned later (at the end of the period), we say that the task has immediate costs. The task has immediate rewards if the reward is earned immediately and the cost is incurred later.

The agent is assumed to have *a quasi-hyperbolic discounting or* $(\beta, \delta)$-*preferences* where $\delta$ represents the long-run discounting, while $\beta \leq 1$ represents the present-bias discounting. Following ODR99, we assume $\delta = 1$ and $\beta < 1$. Specifically, given the agent's present bias $\beta$, her utility value of performing the task in period $\tau \geq t$ when evaluated from the perspective of period $t$ is

$$U^t_\beta(\tau) \equiv \begin{cases} \beta v_\tau - c_\tau & \text{if } \tau = t \\ \beta v_\tau - \beta c_\tau & \text{if } \tau > t \end{cases}$$

if the task has immediate costs, and

$$U^t_\beta(\tau) \equiv \begin{cases} v_\tau - \beta c_\tau & \text{if } \tau = t \\ \beta v_\tau - \beta c_\tau & \text{if } \tau > t \end{cases}$$

if the task has immediate rewards.

Following ODR99, we say that an agent is *a naif* if she believes herself to be time-consistent, *a sophisticate* if she is fully aware of her present-bias. Formally, let $\hat{\beta} \in \{\beta, 1\}$ denote the agent's awareness of her (actual) present-bias discounting factor $\beta < 1$. Note that $\hat{\beta} = 1$ for a naif, and $\hat{\beta} = \beta$ for a sophisticate.

A strategy of an agent is described by a vector $s \equiv (s_1, s_2, \ldots, s_T) \in \{Y, N\}^T$ where $s_t = Y$ if the agent would perform the task in period $t$, $s_t = N$ if she would not. Note that $s_T = Y$ because the agent must complete the task before the end of period $T$. ODR99 argue that the optimal strategy for the agent must be consistent with her belief about how she would do in the future. The so-called perception perfect strategy is determined by evaluating the utility of completing the task now against the utility of postponing it to the earliest future period in which the agent expects to undertake it. Formally, the agent's expectation about herself to undertake the task in the future is described by the following sequence $s^* = (s^*_2, s^*_3, \ldots, s^*_{T-1}, Y)$. Whenever it is necessary, we shall denote it by $s^*(\hat{\beta})$ to show its dependence on $\hat{\beta}$. Moreover, the earliest future period in which the agent expects herself to undertake the task is defined as follows.

**Definition 1** (immediate successor)**.** Let $s^*$ be an agent's future belief. For a fixed $\tau$ and $t < \tau < T$, the immediate successor of $\tau$ is the earliest future date chosen to complete the task after $\tau$ according to $s^*$. Formally,

$$\tau^r = \arg\min\{ t^r : t^r > \tau \text{ and } s^*_{t'} = \mathrm{Y} \}$$

Then, the perception-perfect strategy is defined as follows.

**Definition 2** (perception-perfect strategy)**.** A perception-perfect strategy is a strategy $s^{pp} = (s_1^{pp}, s_2^{pp}, \ldots, s_{T-1}^{pp}, Y)$ which satisfies that for all $t = 1, 2, \ldots, T-1$, $s_t^{pp} = Y$ if and only if

$$U_\beta^t(\tau) \geq U_\beta^t(\tau^r),$$

where $\tau^r$ is the immediate successor of $\tau$ in $s^*(\hat{\beta})$.

The above definition reduces to the definition in ODR99 for the time-consistent if $\beta = 1$, for the naifs if $\hat{\beta} = 1$, and for the sophisticates if $\hat{\beta} = \beta < 1$. The actual task completion date is computed from the agent's perception-perfect strategy $s^{pp}$ as follows:

$$\tau^{pp} = \min\{t : s_t^{pp} = \text{Y}\}.$$

For later use, we define $\tau_{TC}$, $\tau_n$, and $\tau_s$ to be the task completion date of a time-consistent agent, a naif, and a sophisticate, respectively.

## 2.2 Preemptive Overcontrol: Example in O'Donoghue and Rabin (1999)

In this section, we consider Example 3 in O'Donoghue and Rabin (1999) that illustrates "preemptive overcontrol" where a sophisticate preemptively performs a given task earlier than a time-consistent counterpart, $\tau_s < \tau_{tc}$.

**Example 1.** Suppose that costs are immediate, $T = 3$, and $\beta = \frac{1}{2}$ for naifs and sophisticates. Let $\mathbf{v} = (12, 18, 18)$ and $\mathbf{c} = (3, 8, 13)$. Because $\mathbf{v} - \mathbf{c} = (9, 10, 5)$, $\tau_{tc} = 2$ for $v_t - c_t$ is maximized at $t = 2$. A naif believes her perception-perfect strategy to be the one of a time-consistent agent, so her future belief is $(N, Y, Y)$. Then, the naif decides not to perform a given task at $t = 2$, $s_{t=2}^{n} = N$, because

$$\beta v_2 - c_2 = \frac{1}{2} \times 18 - 8 = 1 < \beta(v_3 - c_3) = \frac{1}{2} \times (18 - 13) = 2.5$$

and she decides not to either at $t = 1$, $s_{t=1}^{n} = N$, because

$$\beta v_1 - c_1 = \frac{1}{2} \times 12 - 3 = 3 < \beta(v_2 - c_2) = \frac{1}{2} \times (18 - 8) = 5.$$

Therefore, the naif's perception-perfect strategy is $(N, N, Y)$ thus $\tau_n = 3$. A sophisticate decides not to perform a given task at $t = 2$, $s_{t=2}^{s} = N$, because

$$\beta v_2 - c_2 = \frac{1}{2} \times 18 - 8 = 1 < \beta(v_3 - c_3) = \frac{1}{2} \times (18 - 13) = 2.5$$

but she decides to perform at $t = 1$, $s_{t=1}^{s} = Y$, because

$$\beta v_1 - c_1 = \frac{1}{2} \times 12 - 3 = 3 > \beta(v_3 - c_3) = \frac{1}{2} \times (18 - 13) = 2.5.$$

Then, the sophisticate's perception-perfect strategy is $(Y, N, Y)$ thus $\tau_s = 1$. In all, $\tau_s = 1 < \tau_{tc} = 2 < \tau_n = 3$.

The present bias effect delays performing a given task while the sophistication effect properates it. Propositions 1 and 2 in O'Donoghue and Rabin (1999) show this formally that $\tau_n > \tau_{tc}$ and $\tau_n > \tau_s$. In other words, a naif performs a given task later than a time- consistent agent because of the present-bias effect, but sophistication mitigates such an effect so that a sophisticate performs earlier than a naif.

The preemptive overcontrol as in the above example occurs when the sophistication effect dominates the present-bias effect. When costs are immediate, neither necessary nor sufficient condition is known for the relationship between $\tau_{tc}$ and $\tau_s$.

# 3 What is Required for Preemptive Overcontrol?

In this section, we investigate what is required for preemptive overcontrol, that is, the necessary conditions. A usual understanding about the requirement is that a sophisticate over-reacts to her present bias because she anticipates even a single delay will cause her to finish a given task beyond the ideal date. Formally, this explanation can be written as follows:

**Conjecture 1.** *Suppose that costs are immediate and preemptive overcontrol occurs, $\tau_s < \tau_{tc}$. Then, the immediate successor $\tau^r$ of $\tau_s$ for a sophisticate satisfies $\tau_s < \tau_{tc} < \tau^r$.*

The following example shows that this conjecture does not hold.

**Example 2.** Suppose that costs are immediate. $T = 4$, and $\beta = 1/2$ for naifs and sophisticates. Let $\mathbf{v} = (102, 120, 180, 180)$ and $\mathbf{c} = (5, 30, 80, 130)$. The net reward $\mathbf{v} - \mathbf{c}$ is maximized at $t = 3$ and the perception-perfect strategy of a sophisticate is $(Y, Y, N, Y)$. Then, preemptive overcontrol arises because $\tau_{tc} = 3$ and $\tau_s = 1$. However, a single delay at $\tau_s = 1$ does not lead to procrastination beyond the ideal task-completion date $t = \tau_{tc}$ because $s_2^s = Y$ (a sophisticate would perform a given task at $t = 2$ earlier than $\tau_{tc}$).

This example raises a question as to what conditions are necessary for preemptive over- control and whether they can be related to the usual understanding, if at all. In what follows, we address this question. For simplicity of exposition, we assume that $v_t - c_t$ has a unique maximum, or equivalently, $\tau_{tc}$ is unique.

We first show the condition under which the sophistication effect exactly offsets the present bias effect, that is, a sophisticate completes a task at the ideal task completion date that a time-consistent individual would choose.

**Lemma 1.** *Suppose that* $s^{S}_{\tau_{tc}}=Y$. *Then,* $\tau_{tc}=\tau_{s}$

*Proof.* By definition of $\tau_{tc}$, $\beta(v_t - c_t) < \beta(v_{\tau_{tc}} - c_{\tau_{tc}})$ for any $t < \tau_{tc}$. This implies $s^{S}_{t} = N$ for $t < \tau_{tc}$ because $\beta v_t - c_t < \beta(v_{\tau_{tc}} - c_{\tau_{tc}})$. Since $\tau_{tc}$ is the earliest date at which a sophisticate performs a given task, $\tau_{tc} = \tau_{s}$.

The above lemma shows that if a sophisticate expects herself to perform a task on the optimal date, then she would delay until then. Before presenting our main theorem, we show the restriction on the net reward schedule $\mathbf{d} = \mathbf{v} - \mathbf{c}$ imposed by the perception perfect strategy of a sophisticate.

**Lemma 2.** *Let* $\tau'$ *be the immediate successor of* $\tau$ *for sophisticates. Then, the following must hold:*

$$v_{\tau} - c_{\tau} > v_{\tau'} - c_{\tau'}.$$

*Proof.* Because $\tau'$ is the immediate successor of $\tau$, $\beta(v_{\tau'} - c_{\tau'}) < \beta v_{\tau} - c_{\tau}$. Moreover, $\beta v_{\tau} - c_{\tau} < \beta(v_{\tau} - c_{\tau})$ for $\beta < 1$. In all, $v_{\tau'} - c_{\tau'} < v_{\tau} - c_{\tau}$.

The above lemma implies that the associated net rewards on the dates at which a sophisticate would perform a task decreases in time. Let $d_{t_k}$ be a subsequence of $d_t$ such that $s^{S}_{t_k} = Y$ and its length is less than or equal to $T$. Then, Lemma 2 implies that $d_{t_k}$ is decreasing, that is, $d_{t_k}=v_{t_k} - c_{t_k} > d_{t_{k'}} = v_{t_{k'}} - c_{t_{k'}}$ for any $k$ and $k'$ with $k' > k$.

Now, we provide necessary conditions for preemptive overcontrol.

**Theorem 1.** *Suppose that costs are immediate and* $\tau_{s} < \tau_{tc}$. *Then, the following conditions hold:*

(i) *neither rewards nor costs are constant across all periods,*

(ii) $d_t := v_t - c_t$ *is not monotonic in t, and*

(iii) $s^{S}_{\tau_{tc}} = N$.

The proof for claim (iii) is trivial by Lemma 1. The rest of the proof in the below.

*Proof.* For (i), suppose for contradiction that the reward schedule or the cost schedule is constant, $v_t = \bar{v}$ or $c_t = \bar{c}$. Assume without loss of generality that $v_t = \bar{v}$. By claim (iii), $s^S_{\tau_{tc}} = N$. Let $\tau^r = \min\{t \geq \tau_{tc} : s^S_t = Y\}$ be the earliest date at which the sophisticates would complete a given task when they procrastinate at date $\tau_{tc}$. Obviously, $\tau^r$ exists because $s^S_T = Y$. By definition, $\tau^r$ satisfies

$$\beta_{c_{t'}} < c_{\tau_{tc}} \tag{1}$$

Because $\tau_s < \tau_{tc} < \tau^r$, there exist an integer $K$ and an increasing sequence $(\tau_{(k)})$ of length $K$ such that $\tau_s < \tau_{(1)} < \tau_{(2)} < \cdots < \tau_{(K)} < \tau^r$ and $s^S_{\tau(k)} = Y$ for $k = 1, 2, \ldots, K$. By Lemma 2, we have

$$\bar{v} - c_{\tau_s} > \bar{v} - c_{\tau_{(1)}} > \bar{v} - c_{\tau_{(2)}} > \cdots > \bar{v} - c_{\tau_{(k)}} > \bar{v} - c_{\tau'},$$

that is,

$$c_{\tau_s} < c_{\tau'} \tag{2}$$

Equations (1) and (2) imply $c_{\tau_s} < c_{\tau_{tc}}$, or equivalently, $\bar{v} - c_{\tau_{tc}} < \bar{v} - c_{\tau_s}$. This is a contradiction because the time-consistent agent would be better off completing a given task earlier at date $\tau_s$. The remaining case of constant costs $c_t = \bar{c}$ can be treated similarly.

For claim (ii), suppose for contradiction that $v_t - c_t$ increases monotonically in $t$. Then, it is trivial that $\tau_{tc} = T$. Moreover, observe that for any $t = 1, 2, \ldots, T-1$,

$$\beta v_t - c_t < \beta(v_t - c_t) < \beta(v_T - c_T).$$

Therefore, $\tau_s = T$. This contradicts our assumption that $\tau_s < \tau_{tc}$. Now, suppose for contradiction that $v_t - c_t$ decreases monotonically in $t$. Then, $\tau_{tc} = 1$. Because $\tau_s$ can only take values from 1 to $T$, this contradicts our assumption that $\tau_s < \tau_{tc}$.

The above theorem, Theorem 1, provides a partial answer as to when and whether the sophistication effect dominates the present-bias effect.

Therefore, it complements O'Donoghue and Rabin (1999) by establishing the relationship between $\tau_{tc}$ and $\tau_s$ when costs are immediate.

**Corollary 1.** *Suppose that costs are immediate and the cost and reward schedules satisfy either one of the following:*

*(i)* **c** *or* **v** *is constant; (ii)* **d** *is monotone; (iii)* $s^{s}_{\tau_{tc}}=Y$

*Then, the task completion dates for a time-consistent, a naif, and a sophisticate satisfy*

$$\tau_n \geq \tau_s \geq \tau_{tc}.$$

*That is, preemptive overcontrol does not arise.*

Theorem 1 and Lemma 2 together imply the following.

**Proposition 1.** *Suppose that costs are immediate and* $\tau_s < \tau_{tc}$. *Then, there must exist the immediate successor* $\tau'$ *of* $\tau_s$ *such that a single delay on* $\tau_s$ *must decrease the net reward more than* $(1-\beta)c_{\tau_s}$ *by the next task completion date* $\tau'$, *that is,*

$$d_{\tau_s} - d_{\tau'} > (1-\beta)c_{\tau_s} > 0$$

The proof is trivial because the immediate successor $\tau'$ of $\tau_s$ satisfies $\beta(v_{\tau'} - c_{\tau'}) < \beta v_{\tau_s} - c_{\tau_s} < \beta(v_{\tau_s} - c_{\tau_s})$. The above proposition (Proposition 1) provides why a sophisticate overreacts preemptively to her present bias. The sophisticate fears that if she postpones, the net reward from completing the task will significantly decrease by the next task-completion date. Note that both in Examples 1 and 2, the net reward on the next task-completion date (the immediate successor of $\tau_s$) decreases and its decrement is larger than $(1-\beta)c_{\tau_s}$.

Now, we investigate whether and how our (correct) intuition about preemptive overcontrol can be connected to the usual understanding as formalized in Conjecture 1. Comparison between Examples 1 and 2 reveals that in the former, the net reward schedule is monotonically increasing before $\tau_{tc}$ and then decreasing afterwards. Formally, we assume the following.

**Assumption 1** (Single-Peakedness)**.** The net reward schedule $\mathbf{d} := \mathbf{v} - \mathbf{c}$ is single-peaked, that is, $d_t$ is increasing for $t < \tau_{tc}$ and decreasing for $t > \tau_{tc}$.
If this is the case, Lemma 2 implies that the immediate successor of $\tau_s$ cannot occur earlier than $\tau_{tc}$. Then, Proposition 1 implies Conjecture 1.

**Proposition 2.** *Suppose that* $\mathbf{d}$ *is single-peaked and* $\tau_s < \tau_{tc}$. *Then, Conjecture 1 holds true.*

*Proof.* Suppose for contradiction that $\tau_s < \tau \leq \tau_{tc}$. By Lemma 2, $v_{\tau_s} - c_{\tau_s} > v_{\tau'} - c_{\tau'}$. This contradicts the single-peakedness of $\mathbf{d}$.

This proposition demonstrates that the usual understanding implicitly presumes the single-peakedness of the net reward schedule.

# 4 Concluding Remarks

In this paper, we study the conditions required for preemptive overcontrol, that is, an agent's sophistication results in an overly preemptive measure of completing a given task prematurely earlier than the ideal date. The necessary conditions reveal that the usual understanding about preemptive overcontrol is incorrect. A sophisticate preemptively overreacts to her present bias not because (i) she anticipates even a single delay will lead to procrastination beyond the ideal date, but because (ii) she anticipates such a delay will lead to a significant decrease in the net reward of completing a task by the next completion date. We suggest that single-peakedness of the net reward schedule might be a culprit for yielding the incorrect usual understanding of preemptive overcontrol (explanation (i) above).

## References

Augenblick, N., M. Niederle, and C. Sprenger, "Working over time: Dynamic inconsistency in real effort tasks," *The Quarterly Journal of Economics*

130, 2015, 1067–1115.

Augenblick, N. and M. Rabin, "An experiment on time preference and misprediction in unpleasant tasks," *Review of Economic Studies* 86, 2019, 941–975.

Bisin, A. and K. Hyndman, "Present-bias, procrastination and deadlines in a field experi- ment," *Games and Economic Behavior* 119, 2020, 339–357.

Ericson, K. M. and D. Laibson, "Intertemporal choice": In Bernheim, B. D., S. DellaVigna, and D. Laibson eds., *Handbook of Behavioral Economics: Applications and Foundations* 2, North-Holland, 2019, 1–67.

Frederick, S., G. Loewenstein, and T. O'Donoghue, "Time discounting and time preference: A critical review," *Journal of Economic Literature* 40, 2002, 351–401.

O'Donoghue, T. and M. Rabin, "Doing It Now or Later," *American Economic Review* 89, 1999, 103–124.