Indeterminacy, maintenance expenditures and capital-labour substitution elasticity

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Abstract

Recent research in macroeconomics has focused on the possibility of indeterminacy since a model with indeterminacy can generate business cycles that are driven by sunspots. However, most of existing works in this area build on the Cobb-Douglas production function, which seems not empirically plausible. This paper departs from unitary values for the elasticity of capital-labour substitution in production and investigates the role of capital-labour substitution elasticity on the occurrence of indeterminacy. I show that in a two-sector model with maintenance expenditures, the minimum degree of externalities required for indeterminacy is much lower than its one-sector predecessor. Moreover, results show that indeterminacy may still occur when the elasticity of labour supply is within the range of empirical plausibility.

Keywords: Indeterminacy, Capital-Labour Substitution, Maintenance Expenditures

JEL Classification: E32

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1 Introduction

Research in macroeconomics has focused on the possibility of indeterminacy since models with indeterminacy can generate business cycles that are driven by agents' self-fulfilling beliefs. Beginning with Benhabib and Farmer's (1994) one-sector model, theoretical literature in this area has shown that a certain degree of increasing returns, often exhibited via external effects, must be present in order to generate multiple equilibria. However, empirical studies such as Basu and Fernald (1997) have concluded that the returns to scale are roughly constant, which has stimulated researchers to pursue model structures with lower scale economies and triggered the researchers' concern about the empirical plausibility of indeterminacy models.

Remarkable progress has been made in indeterminacy models in terms of empirical plausibility. There are several ways to produce indeterminacy with increasing returns to scale that are lower than early studies: (1) consider two-sector economies with sector-specific externalities or multi-sector models such as Benhabib and Farmer (1996), Benhabib and Nishimura (1998); (2) add capital utilization into the production function such as Wen (1998), Guo and Harrison (2001); (3) use different specification of functions such as Guo and Lansing (2007).

It is a well-known fact that the elasticity of substitution between capital and labour is an important determinant of economic growth in the neoclassical growth model.[1] There is no room for discussing the role of elasticity of capital-labour substitution if it is simply assumed to be unitary. In fact, there are mounting empirical studies against unitary substitution elasticity, for example, Arrow et al. (1961), Berndt (1976) and Antras (2004) to name just three. Although the empirical estimates of the elasticity of capital-labour substitution are not robust, most studies support the view that the elasticity is significantly below unity and ranges from 0.4 to 0.6 for the US[2].

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[1] For example, Solow (1956), De La Grandville (1989) and Klump and De La Grandville (2000). However, all of above research restrict their attention to a Cobb-Douglas specification which has been widely used as default production function in business cycle literature.
[2] Wong and Yip (2010) provide a summary of the estimated substitution elasticities based on US data in the literature.

In light of these issues, Pintus (2006) has explored a one-sector Ramsey model and concludes that indeterminacy may occur with small degree of increasing returns when the elasticity of capital-labour substitution is large enough ([2.16,13.37]), given that the labour supply is close to indivisible, which appears to be inconsistent with empir- ical estimates[3]. More recent works such as Guo and Lansing (2009) and Photphisutthiphong and Weder (2012) show that an elasticity of capital-labour substitution below unity may not rule out indeterminacy. However, a higher degree of increasing returns is required to produce sunspot equilibria. Moreover, these models assume infinite elasticity of labour supply, which is far away from empirical observations. Keane and Rogerson (2012) demonstrate that the labour supply elasticity at the macro level falls in the range of 1 to 2.

This paper considers a two-sector version of Guo and Lansing's (2009) model in which depreciation rate depends on both capital utilization and maintenance expenditures, motivated by the fact that capital maintenance expenditures account for a large fraction of new investment and output, indicating that these expenditures are 'too big to ignore'[4]. I show that in the two-sector model, the minimum degree of externalities needed to produce indeterminacy is much lower than its one-sector predecessor. For example, when the elasticity is 0.6, the threshold returns to scale could be as small as 1.0263. Moreover, I show that the required returns to scale for multiple equilibria is still empirically plausible even if the elasticity of labour supply is lowered to 1 or 2.

The intuition of indeterminacy can be formulated as follows: when agents are optimistic about the future and believe that the rate of return on capital will increase, they increase their investment goods expenditure and reallocate factors between sectors, increasing output of investment goods sector and therefore increasing the future capital stock. Due to externality, both marginal product of capital and marginal product of labour increase in the investment sector. Meanwhile, when the relative price of investment goods decreases, then agents' expectation can be self-fulfilling. On the one hand, lower factor substitutability makes the

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[3] Wong and Yip (2010) study the stability properties of one-sector economies with an elasticity of substitution between capital and labour that is below unity.

[4] McGrattan and Schmitz (1999) define maintenance expenditures as "the expenditures made for the purpose of keeping the stock of fixed assets or productive capacity in good working order during the life originally intended".

factor reallocation between sectors more difficult and thus higher returns to scale are required to produce sunspot equilibria. On the other hand, maintenance activities make the existing capital more productive and thus boost the net rate of return on capital (Jiang 2017), which offsets the effect of lower factor substitution elasticity.

The rest of this paper is organized as follows. Section 2 presents the two-sector model and analyzes the local dynamics. Section 3 uses numerical methods to derive the conditions for indeterminacy under different parameterizations and discusses the intuitions. Section 4 concludes.

2 The model

This paper considers a two-sector version of Guo and Lansing (2009)'s model which incorporates two main features: (1) the capital-labour substitution elasticity is less than one; (2) capital maintenance expenditures are considered. There are two sectors in the economy: consumption sector and investment sector. Firms can either produce consumption good or investment good. Households own the firms and make decisions on capital maintenance expenditure.

2.1 Households

A representative household chooses the sequences of consumption ct and hours worked l_t to maximize his lifetime utility

$$\Sigma_{t=0}^{\infty} E_t \beta^t \left(log\ c_t - \frac{l_t^{1+\chi}}{1+\chi} \right) \quad (1)$$

subject to the budget constraint

$$c_t + p_t i_t = r_t u_t k_t + w_t l_t, \quad (2)$$

where $1/\chi$ stands for the labour supply elasticity. β is a discount factor implying that future consumption is less valuable than current consumption.

i_t is the household's investment in new capital. p_t is the relative price of investment goods in units of consumption goods. r_t and w_t are the rental rate of capital and the real wage rate which individuals and firms take as given. u_t is the rate of capital utilization. Let k_t denote capital stock. The law of motion for capital accumulation is then given by

$$k_{t+1} = i_t + (1-\delta_t)k_t - m_t, \tag{3}$$

where m_t represents goods expenditures on maintenance activities. Following Guo and Lansing (2009), the depreciation rate of the capital stock δ_t is increasing with capital utilization u_t and decreasing with maintenance expenditures

$$\delta_t = \tau \frac{u_t^{\theta}}{(m_t/k_t)^{\phi}}, \tau > 0, \theta > 1, \phi \geqslant 0, \tag{4}$$

where θ and φ are the elasticities of depreciation rate with respect to capital utilization and maintenance cost rate m_t/k_t, respectively.

The Lagrangian setup for household is given by

$$L = \Sigma_{t=0}^{\infty}\beta^t \left\{\left(ln\, c_t - \frac{l_t^{1+\chi}}{1+\chi}\right) + \lambda_t[(r_t u_t k_t + w_t l_t - c_t)p_t^{-1} + (1-\delta_t)k_t - m_t - k_{t+1}]\right\} \tag{5}$$

where λ_t is the Lagrange multiplier. The corresponding first order conditions are

$$\frac{1}{c_t} = \lambda_t p_t^{-1}, \tag{6}$$

$$\hat{\lambda}_t w_t p_t^{-1} = l_t^{\chi}, \tag{7}$$

$$r_t p_t^{-1} u_t = \theta \delta_t, \tag{8}$$

$$\phi \delta_t \frac{k_t}{m_t} = 1, \tag{9}$$

$$k_{t+1} = (r_t u_t k_t - w_t l_t - c_t)p_t^{-1} + (1-\delta_t)k_t - m_t, \tag{10}$$

$$\lambda_t = \beta E_t \lambda_{t+1}[(r_{t+1}u_{t+1}p_{t+1}^{-1}) - (\phi+1)\delta_{t+1} + 1]. \tag{11}$$

2.2 Firms

In the consumption sector, firms maximize their profit using the following CES production function

$$y_{ct} = A\left[\alpha(u_t k_{ct})^{\varphi} + (1-\alpha)l_{ct}^{\varphi}\right]^{\frac{1}{\varphi}} Y_{ct}^{\frac{\eta}{1+\eta}}, A > 0, \eta \geqslant 0 \tag{12}$$

where $\varphi = \frac{\sigma-1}{\sigma} \cdot \sigma$ refers to the elasticity of capital-labour substitution. α is the share parameter. y_{ct}, k_{ct} and l_{ct} are the firm-level of output of consumption goods, capital and labour used in the consumption sector. $Y_{ct}^{\frac{\eta}{1+\eta}}$ captures the external effects where Y_{ct} is the economy-wide average output level and η denotes the degree of sector-specific externalities.

In the investment sector, firms maximize profit subject to

$$y_{it} = B\left[\alpha(u_t k_{it})^{\varphi} + (1-\alpha)l_{it}^{\varphi}\right]^{\frac{1}{\varphi}} Y_{it}^{\frac{\eta}{1+\eta}}, B > 0 \tag{13}$$

The variables are defined as above, but for the investment sector[5]. Terms A and B are efficiency parameters.

I assume that the capital-labour substitution elasticities are the same in the two sectors so that the results are comparable with existing literature. Moreover, most empirical studies report estimates of capital-labour substitution elasticities for the aggregate economy and I didn't find evidence that shows systematic difference between the sectors.

Assuming that the factor markets are perfectly competitive in both sectors, demands for capital and labour satisfy the following conditions:

[5] Harrison (2001) demonstrates that the indeterminacy properties are independent of the degree of the returns to scale in the consumption sector, providing that the utility function is logarithmic in consumption. Therefore in this paper I assume that the size of externalities in two the sectors are the same, η.

$$u_t r_t = \frac{\alpha y_{ct}(u_t k_{ct})^{\varphi}}{k_{ct}[\alpha(u_t k_{ct})^{\varphi}+(1-\alpha)l_{ct}^{\varphi}]} = p_t \frac{\alpha y_{it}(u_t k_{it})^{\varphi}}{k_{it}[\alpha(u_t k_{it})^{\varphi}+(1-\alpha)l_{it}^{\varphi}]} \quad (14)$$

$$w_t = \frac{(1-\alpha) y_{ct} l_{ct}^{\varphi}}{l_{ct}[\alpha(u_t k_{ct})^{\varphi}+(1-\alpha)l_{ct}^{\varphi}]} = p_t \frac{(1-\alpha) y_{it} l_{it}^{\varphi}}{l_{it}[\alpha(u_t k_{it})^{\varphi}+(1-\alpha)l_{it}^{\varphi}]} \quad (15)$$

2.3 Equilibrium and dynamics

This paper focuses on symmetric equilibrium. Let s_t be the share of total capital and labour used in the consumption sector

$$s_t = \frac{k_{ct}}{k_t} = \frac{l_{ct}}{l_t} \ . \quad (16)$$

Symmetric equilibrium requires that all firms take same actions. Quantities demanded by households and supplied by firms in the consumption sector and investment sector are equal. Then the following must hold:

$$c_t = y_{ct} = Y_{ct},\ i_t = y_{it} = Y_{it}. \quad (17)$$

Moreover, the factor market clears. Total production in the economy consists of consumption and investment goods produced in their own sectors:

$$k_{ct} + k_{it} = k_t,\ l_{ct} + l_{it} = l_t. \quad (18)$$

$$y_{ct} + y_{it} = y_t, \quad (19)$$

implying that

$$y_t = A^{1+\eta} s_t^{\eta} \left[\alpha(u_t k_t)^{\varphi} + (1-\alpha) l_t^{\varphi}\right]^{\frac{1+\eta}{\varphi}} \quad (20)$$

Substituting Equations (16)–(19) into (14)–(15), one can obtain the following factor prices and the price of investment good relative to consumption good:

$$u_t r_t = \frac{\alpha y_t (u_t k_t)^{\varphi}}{k_t\left[\alpha(u_t k_t)^{\varphi}+(1-\alpha)l_t^{\varphi}\right]} \tag{21}$$

$$w_t = \frac{(1-\alpha) y_t l_t^{\varphi}}{l_t\left[\alpha(u_t k_t)^{\varphi}+(1-\alpha)l_t^{\varphi}\right]} \tag{22}$$

$$p_t = \left(\frac{A}{B}\right)^{1+\eta}\left(\frac{s_t}{1-s_t}\right)^{\eta} \tag{23}$$

The dynamics of this economy are summarized by the equations (6) – (11) and (20) – (23).

This paper analyses the effect of a change in capital-labour substitution elasticity on the existence of equilibrium indeterminacy. For this purpose, the CES production functions are normalized[6] to maintain the steady-state values in the Cobb-Douglas case where $\sigma = 1$ as the elasticity of capital-labour substitution varies[7] (Guo and Lansing 2009). Then I log-linearize the system of equations around the normalized steady state.

I use a bar over parameters and variables to denote the steady-state values from the Cobb-Douglas case. The following steady-state quantities of the Cobb-Douglas case can be computed directly from Jiang (2017):

$$\bar{s} = \frac{\theta-\bar{\alpha}(1+\phi)}{\theta} \tag{24}$$

$$\bar{\delta} = \frac{1-\beta}{\beta(\theta-\phi-1)} \tag{25}$$

$$\frac{\bar{m}}{\bar{k}} = \phi\bar{\delta} \tag{26}$$

$$\bar{u} = \left\{\left[\frac{(1-\bar{s})(1-\beta)\theta}{\bar{\alpha}\beta(\theta-\phi-1)} - \frac{\bar{m}}{\bar{k}}\right]\left(\frac{\bar{m}}{\bar{k}}\right)^{\phi}\right\}^{\frac{1}{\theta}} \tag{27}$$

$$\bar{k} = \left[\frac{\bar{\delta}(1+\phi)}{(1-\bar{s})\bar{u}^{\bar{\alpha}(1+\eta)}\bar{l}^{(1-\bar{\alpha})(1+\eta)}}\right]^{\frac{1}{\bar{\alpha}(1+\eta)-1}} \tag{28}$$

[6] In Klump and De La Grandville (2000), Klump and Saam (2008), the use of normalized CES function is introduced.

[7] By normalization, one can work with family of CES functions that are distinguished by their elasticity parameters only (Klump & De La Grandville 2000).

where the capital share of income for the Cobb-Douglas case $\overline{\alpha}$ is set to 0.35, the discount factor β equals 0.99[8]. As Guo and Lansing (2009), the parameters θ and ϕ are set to maintain the following calibrations used in the Cobb-Douglas case as σ changes: $\overline{m}/\overline{y} = 0.061$, $\overline{\delta} = 0.025$ and $\overline{l} = 0.3$, implying that $\phi = 0.2955$ and $\theta = 1.6955$. These parameter values are consistent with US data and are common in the real business cycle literature.

The distribution parameter α in the model is dependent on σ and is set to maintain the value of $\overline{\alpha}$. The efficient parameters A and B are set to maintain the steady-state value of output equals to $\bar{y}$:

$$\alpha = \frac{\bar{\alpha}}{\bar{\alpha}+(1-\bar{\alpha})\left(\frac{\bar{u}\bar{k}}{\bar{l}}\right)^{\varphi}} \tag{29}$$

$$A = \frac{(\bar{s}\bar{y})^{\frac{1}{1+\eta}}}{\bar{s}[\alpha(\bar{u}\bar{k})^{\varphi}+(1-\alpha)\bar{l}^{\varphi}]^{\frac{1}{\varphi}}} \tag{30}$$

$$B = \frac{[(1-\bar{s})\bar{y}]^{\frac{1}{1+\eta}}}{(1-\bar{s})[\alpha(\bar{u}\bar{k})^{\varphi}+(1-\alpha)\bar{l}^{\varphi}]^{\frac{1}{\varphi}}} \tag{31}$$

Let hat variables denote percentage deviation from their steady-state values. Let $\frac{\alpha(\bar{u}\bar{k})^{\varphi}}{\alpha(\bar{u}\bar{k})^{\varphi}+(1-\alpha)\bar{l}^{\varphi}} = \Theta$, $\frac{(1-\alpha)(\bar{l})^{\varphi}}{\alpha(\bar{u}\bar{k})^{\varphi}+(1-\alpha)\bar{l}^{\varphi}} = \Omega$ and

$$\mathbb{Y}_t = \begin{pmatrix} \hat{s}_t \\ \hat{y}_t \\ \hat{u}_t \\ \widehat{m}_t \\ \hat{l}_t \\ \hat{r}_t \\ \widehat{w}_t \\ \hat{p}_t \end{pmatrix}, \mathbb{S}_t = \begin{pmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{pmatrix}$$

Then the model writes:

[8] Let τ be 1 as it does not affect the results.

$$\Pi_1 \mathbb{Y}_t = \Pi_2 \mathbb{S}_t \tag{32}$$

$$\Gamma_1 \mathbb{S}_{t+1} + \Gamma_2 \mathbb{Y}_{t+1} = \Gamma_3 \mathbb{S}_t + \Gamma_4 \mathbb{Y}_t \tag{33}$$

where

$$\Pi_1 = \begin{pmatrix} -\eta & 1 & -(1+\eta)\Theta & 0 & -(1+\eta)\Omega & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -\chi & 0 & 1 & -1 \\ 0 & 0 & \theta & -(\phi+1) & 0 & 0 & 0 & 0 \\ 0 & 0 & -(\theta-1) & \phi & 0 & 1 & 0 & -1 \\ 0 & -1 & 1-\varphi+\varphi\Theta & 0 & \varphi\Omega & 1 & 0 & 0 \\ 0 & -1 & \varphi\Theta & 0 & 1-\varphi+\varphi\Omega & 0 & 1 & 0 \\ -\dfrac{\theta\eta}{\bar{\alpha}(1+\phi)} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Pi_2 = \begin{pmatrix} (1+\eta)\Theta & 0 \\ 0 & -1 \\ 0 & -1 \\ -(\phi+1) & 0 \\ \phi & 0 \\ -(1-\varphi+\varphi\Theta) & 0 \\ -\varphi\Theta & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Gamma_1 = \begin{pmatrix} -\dfrac{(1-\beta)(\phi+1)\phi}{\theta-\phi-1} & 1 \\ 1 & 0 \end{pmatrix}$$

$$\Gamma_2 = \begin{pmatrix} 0 & 0 & -\dfrac{(1-\beta)\theta\phi}{\theta-\phi-1} & \dfrac{(1-\beta)(\phi+1)\phi}{\theta-\phi-1} & 0 & \dfrac{(1-\beta)\theta}{\theta-\phi-1} & 0 & -\dfrac{(1-\beta)\theta}{\theta-\phi-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Gamma_3 = \begin{pmatrix} 0 & 1 \\ \dfrac{\beta(\theta-\phi-1)-(1-\beta)(1+\phi)}{\beta(\theta-\phi-1)} & 0 \end{pmatrix}$$

$$\Gamma_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\dfrac{(1-\beta)[\theta-\bar{\alpha}(1+\phi)]}{\beta\bar{\alpha}(\theta-\phi-1)} & \dfrac{(1-\beta)(1+\phi)}{\beta(\theta-\phi-1)} & -\dfrac{(1-\beta)\theta}{\beta(\theta-\phi-1)} & 0 & 0 & 0 & 0 & -\dfrac{(1-\beta)(1+\phi)}{\beta(\theta-\phi-1)} \end{pmatrix}$$

The system boils down to

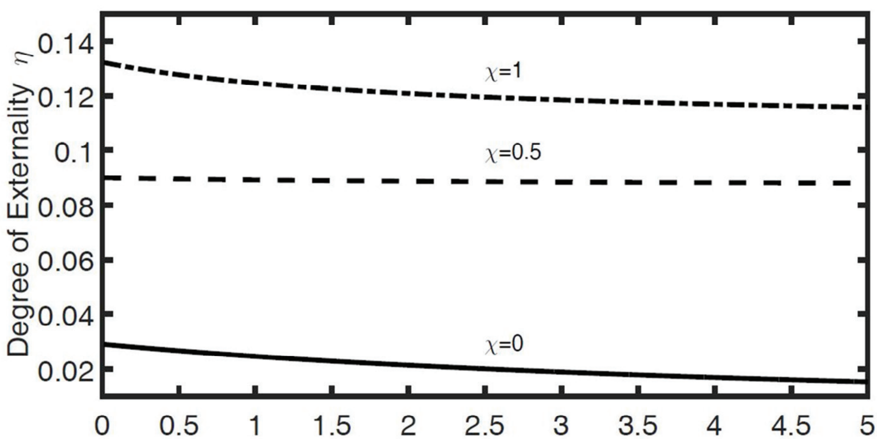
$$\mathbb{S}_{t+1} = J\mathbb{S}_t \tag{34}$$

where J is a 2×2 Jacobian matrix and $J = [\Gamma_1 + \Gamma_2(\Pi_1^{-1}\Pi_2)]^{-1}[\Gamma_3 + \Gamma_4(\Pi_1^{-1}\Pi_2)]$.. Indeterminacy requires that both roots of matrix J are inside the unit circle. Equilibrium conditions and their log- linearizations around the deterministic steady states are given in the Appendix B. In this study I present numerical results only as analytical expressions of trace and determinant of J are too incommodious.

3 Quantitative analysis

Now I investigate the link between the elasticity of capital-labour substitution, maintenance expenditures and the threshold degree of externalities required to generate multiple equilibria. First I restrict $\chi = 0$, implying that the value of the labour supply elasticity is infinite (Hansen's indivisible labour). Figure 1 summarizes the values of η associated with different values of σ. It is evident that the model requires lower minimum degree of returns to scale as the capital-labour substitution elasticity increases.

Figure 1. Elasticity of Capital-Labour Substitution σ



In the extreme case that $\sigma \to 0$ (Leontief case) the minimum degree of production externalities needed for indeterminacy is 0.0290. When Cobb-Douglas production function is assumed, $\sigma = 1$, the model requires that $\eta_{min} = 0.0247$, coinciding with the result of the discrete time version of Jiang's (2017) model. As emphasized earlier, an empirical study by Chirinko (2008) finds that the preferred US estimate of elasticity of capital-labour substitution is well below unity (between 0.4 and 0.6), raising the degree of externalities required to generate multiple equilibria, that is, 0.0272 for $\sigma = 0.4$ and 0.0263 for $\sigma = 0.6$. The numerical results show that the model introduced in this paper can still generate indeterminacy at modest returns to scale.

The intuition for the result is straightforward. Suppose an agent believes that the rate of return on capital will increase in the next period, accordingly the agent increases the demand for investment good. Labour shifts from consumption good sector to investment good sector. As a result, the next period's capital stock will increase. In order that the agent's expectation is supported in the new equilibrium, the capital rental rate must increase at higher level of investment or capital stock, with the first order conditions still holding. This is achievable when sufficient degree of increasing returns are present. Lower elasticity of capital-labour substitution requires that scale economies be higher because lower substitutability between factors makes labour movements across sectors respond more sluggishly to their expectation (higher interest rate). However, the existence of maintenance expenditures reduces the threshold ruturn to scale as well-maintained capital can be more productive and thus boost the net rate of return on capital (Jiang 2017). When maintenance- to-output ratio is set to 0, then $\eta_{min} = \{0.0311, 0.0299, 0.0277\}$ if $\sigma = \{0.4, 0.6, 1\}$.

Although the assumption of $\chi = 0$ (infinite labour supply elasticity) is widely used in early indeterminacy literature beginning with Benhabib and Farmer (1994), empirical evidence[9] indicate that the labour supply elasticity has much lower values. Therefore I also evaluate the stability properties of the two-sector models using parameters that are consistent with US facts. Figure 1 and 2 also show the parameter constellation for indeterminacy when $\chi = 0.5$ and $\chi = 1$ as Keane and Rogerson (2012)

[9] For example, Depew and Sørensen (2013), French and Stafford (2017).

demonstrate that the labour supply elasticity at the macro level falls in the range of 1 to 2. These figures depict that lower elasticity of labour supply increases the threshold values of η_{min}. This follows because the change of this elasticity also affects the mobility of labour across sectors. The results show that indeterminacy can occur in this model with empirically plausible values for maintenance-to- output ratio, labour supply elasticity, capital-labour substitution elasticity and returns to scale.

4 Conclusion

This study introduces an CES production function into two-sector models with maintenance expenditures. It is shown that the smaller the elasticity of capital-labour substitution, the higher degree of externalities for sunspot to occur. However, higher maintenance-to-output ratio reduces the threshold externalities. Using elasticity parameter whose value is ranged from 0.4 to 0.6, which is considered to be empirically plausible, I show that in the model abstracting from Cobb-Douglas technology and infinite elasticity of labour supply, indeterminacy can still occur.

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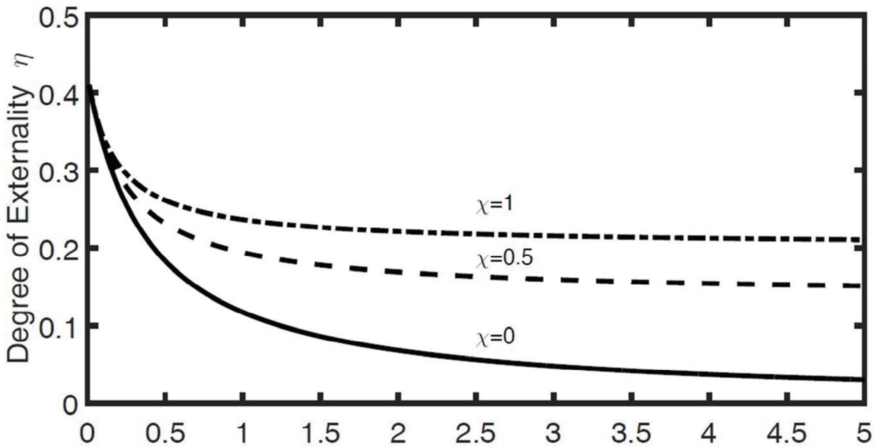
Appendix

Figure 2. Elasticity of Capital-Labour Substitution σ



A. Constant capital utilization

Figure 2 shows the combination of capital-labour substitution elasticity and externality parameters when $u_t = u$. Although the model with constant capital utilization gives the same conclusion that indeterminacy is more difficult to obtain for lower level of substitution elasticity. But it turns out that indeterminacy is no longer possible to occur for the same realistic value of σ. For instance, when $\sigma = \{0, 4, 0.6\}$, the model requires $\eta_{min} = \{0.2066, 0.1664\}$ that is outside the estimates of Basu and Fernald (1995). But the value of the elasticity of substitution at unity only requires that $\eta_{min} = 0.1171$ which is considered to be realistic. The results imply that this model requires a higher degree of externalities than the corresponding model with variable capital utilization rate to generate multiple equilibria as Wen's (2009) so-called "returns-to-scale" effect induced by capital utilization disappeared. It is notable that the slope of the $\sigma - \eta$ curve is steeper for constant capital utilization case as the movement of labour is now coupled with the stock of capital k_t instead of utilized capital $u_t k_t$.

B. Model equations

Appendix B lists the model equilibrium conditions, log-linearized system and derivation of the model's local stability property.

B.1 Equilibrium conditions

$$\frac{1}{c_t} = \lambda_t p_t^{-1}$$

$$\lambda_t w_t p_t^{-1} = l_t^{\chi}$$

$$\phi \tau u_t^{\theta} k_t^{\phi+1} m_t^{-\phi-1} = 1$$

$$r_t p_t^{-1} = \theta \tau u_t^{\theta-1} k_t^{\phi} m_t^{-\phi}$$

$$c_t = A^{1+\eta} s_t^{1+\eta} \left[\alpha (u_t k_t)^{\varphi} + (1-\alpha) l_t^{\varphi}\right]^{\frac{1+\eta}{\varphi}}$$

$$u_t r_t = \frac{\alpha y_t (u_t k_t)^{\varphi}}{k_t \left[\alpha (u_t k_t)^{\varphi} + (1-\alpha) l_t^{\varphi}\right]}$$

$$w_t = \frac{(1-\alpha) y_t l_t^{\varphi}}{l_t \left[\alpha (u_t k_t)^{\varphi} + (1-\alpha) l_t^{\varphi}\right]}$$

$$p_t = \left(\frac{A}{B}\right)^{1+\eta} \left(\frac{s_t}{1-s_t}\right)^{\eta}$$

$$k_{t+1} = (1-s_t) y_t p_t^{-1} + k_t - \tau u_t^{\theta} m_t^{-\phi} k_t^{\phi+1} - m_t$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left[(r_{t+1} u_{t+1} p_{t+1}^{-1}) - (\phi+1) \tau u_{t+1}^{\theta} m_{t+1}^{-\phi} k_{t+1}^{\phi} + 1\right]$$

B.2 Log-linearized system

$$\hat{y}_t = \eta \hat{s}_t + \alpha(1+\eta) \frac{(\bar{u}\bar{k})^{\varphi}}{\alpha(\bar{u}\bar{k})^{\varphi} + (1-\alpha)\bar{l}^{\varphi}} \hat{u}_t + \alpha(1+\eta) \frac{(\bar{u}\bar{k})^{\varphi}}{\alpha(\bar{u}\bar{k})^{\varphi} + (1-\alpha)\bar{l}^{\varphi}} \hat{k}_t + (1-\alpha)(1+\eta) \frac{(\bar{l})^{\varphi}}{\alpha(\bar{u}\bar{k})^{\varphi} + (1-\alpha)\bar{l}^{\varphi}} \hat{l}_t$$

$$\hat{s}_t + \hat{y}_t = -\hat{\lambda}_t + \hat{p}_t$$

$$\hat{\lambda}_t + \widehat{w}_t - \hat{p}_t = \chi \hat{l}_t$$

$$\theta \hat{u}_t + (\phi + 1)\hat{k}_t - (\phi + 1)\widehat{m}_t = 0$$

$$\hat{r}_t - \hat{p}_t = (\theta - 1)\hat{u}_t + \phi \hat{k}_t - \phi \widehat{m}_t$$

$$\widehat{r_t} = \widehat{y_t} - \left[1 - \varphi + \frac{\alpha\varphi(\bar{u}\bar{k})^{\varphi}}{\alpha(\bar{u}\bar{k})^{\varphi} + (1-\alpha)\bar{l}^{\varphi}}\right]\widehat{u_t} - \left[1 - \varphi + \frac{\alpha\varphi(\bar{u}\bar{k})^{\varphi}}{\alpha(\bar{u}\bar{k})^{\varphi} + (1-\alpha)\bar{l}^{\varphi}}\right]\hat{k}_t - \frac{(1-\alpha)\varphi\bar{l}^{\varphi}}{\alpha(\bar{u}\bar{k})^{\varphi} + (1-\alpha)\bar{l}^{\varphi}}\hat{l}_t$$

$$\widehat{w}_t = \hat{y}_t - \left[1 - \varphi + \frac{(1-\alpha)\varphi\bar{l}^{\varphi}}{\alpha(\bar{u}\bar{k})^{\varphi} + (1-\alpha)\bar{l}^{\varphi}}\right]\hat{l}_t - \frac{\alpha\varphi(\bar{u}\bar{k})^{\varphi}}{\alpha(\bar{u}\bar{k})^{\varphi} + (1-\alpha)\bar{l}^{\varphi}}\hat{u}_t - \frac{\alpha\varphi(\bar{u}\bar{k})^{\varphi}}{\alpha(\bar{u}\bar{k})^{\varphi} + (1-\alpha)\bar{l}^{\varphi}}\hat{k}_t$$

$$\hat{p}_t = \frac{\theta\eta}{\bar{\alpha}(1+\phi)}\hat{s}_t$$

$$E_t\hat{\lambda}_{t+1} = \hat{\lambda}_t - \frac{(1-\beta)\theta}{(\theta-\phi-1)}\hat{r}_{t+1} + \frac{(1-\beta)\theta\phi}{\theta-\phi-1}\hat{u}_{t+1} + \frac{(1-\beta)\theta}{(\theta-\phi-1)}\hat{p}_{t+1} - \frac{(1-\beta)(\phi+1)\phi}{(\theta-\phi-1)}\widehat{m}_{t+1} + \frac{(1-\beta)(\phi+1)\phi}{(\theta-\phi-1)}\hat{k}_{t+1}$$

$$\hat{k}_{t+1} = \frac{(1-\beta)(1+\phi)}{\beta(\theta-\phi-1)}\hat{y}_t - \frac{(1-\beta)[\theta-\bar{\alpha}(1+\phi)]}{\beta\bar{\alpha}(\theta-\phi-1)}\hat{s}_t - \frac{(1-\beta)(1+\phi)}{\beta(\theta-\phi-1)}\hat{p}_t + \frac{\beta(\theta-\phi-1)-(1-\beta)(1+\phi)}{\beta(\theta-\phi-1)}\hat{k}_t - \frac{(1-\beta)\theta}{\beta(\theta-\phi-1)}\hat{u}_t$$