Economic effects of strategic trade policy: Focused on the distribution of firms in the 3rd country market

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Abstract

This paper examines the welfare effect of strategic trade policy where firms enter the 3rd country market. The main results are as follows. First, for the case where the number of country 1's firms is one more than that of country 2's, government 1 chooses no subsidy and government 2 adopts subsidy. In this equilibrium, the country 1's welfare decreases and country 2's welfare increases relative to the initial state. Second, for the situation where the number of country 1's firms is higher than the country 2's by more than 1, the government 1 adopts tax and government 2 adopts subsidy. And the welfare of country 1 goes down and that of country 2 rises in this equilibrium compared to the initial state. In this case, the government 1's policy intervention raises the welfare of the country 2, regardless of whether the government 2 intervenes or not.

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1 Introduction

There are many cases that domestic firms export goods to the 3rd country, where they compete with other foreign firms as the economy becomes more globalized. In this case, each country's government may consider policy supports in order for domestic firms to be in a superior position in the competition. Among these kinds of policies, a strategic trade policy has got a lot of attention.

Representative research papers on strategic trade policy are Brander and Spencer (1985), Krugman (1986) and the main contents of these papers are as follows. In the above research, two firms (firm 1, 2) that belong to two countries (country 1, 2) are involved in Cournot competition in the 3rd country market. In this case, if only country 1's government supports subsidy to the firm 1, then firm 1's reaction curve shifts to the right.[1] As a result, firm 1's output and profit increases, and firm 2's output and profit decreases.[2] Next, if both governments support subsidies to both firms, reaction curves of both firms move to the right. As a consequence, both firms' outputs go up and both firms' net profits go down. But since equilibrium strategies of each government is to adopt subsidy support, there occurs the phenomenon of 'prisoner's dilemma.'

The fact that a government can induce the relevant firm to be in the advantageous position in the competition by its subsidy, has the same feature with 'managerial incentive contract' analyzed in Freshtman and Jude (1984). In their paper, the contract between an owner and a manager has the commitment effect in the managerial behavior. Similar to this, a government's subsidy to a firm has the role of providing information to the other firm about the direction of output choice of the sponsored firm.[3]

There is some criticism about the 'strategic trade policy model' be-

[1] From the viewpoint of the firm that receives a subsidy, the effective marginal cost decreases, which leads to the shift of the reaction curve to the right.

[2] In the duopoly market where two firms are involved in Cournot competition, this result comes from the fact that reaction curves in the output space slope downward.

[3] Also, the effect of subsidy is similar to the preemptive investment in Dixit (1980). Namely, in Dixit the incumbent firm's investment changes the equilibrium because the marginal cost of that firm becomes lower. The subsidy support indicates a similar effect as in Dixit.

cause the research results are sensitive to the competition mode of the oligopoly market. For example, Eaton and Grossman (1986) have derived the result that equilibrium strategies of governments are not to support subsides, but to impose export taxes under the Bertrand competition.[4] The reason why this kind of result has occurred is that the price has the characteristics of 'strategic complement.'[5]

In Brander and Spencer (1985), they examine the situation where two firms that belong to two countries compete in the 3rd country market. In contrast, this paper would like to analyze the economic effect of strategic trade policy of each government for the case where there are many firms that belong to each country. Therefore we examine the direction and effect of strategic trade policy for the asymmetric cases where the number of firms that belong to each country is different.

There are also research papers such as Dixit (1984), Krishna-Thursdby (1991) that the results of Brander and Spence model are sensitive to the number of firms. These researches did not derive each firm's profit and each country's welfare change explicitly in the policy choice equilibrium for the various market structures, in comparison with the initial state. In this paper, we specifically analyze the direction of each countries' welfare change by comparing the new equilibrium in the policy choice game for the general oligopoly structure with the initial pre-policy state.

In this paper, I examine the situation where each government introduces subsidy/tax policy to its firms for the case where each country's firms enter the 3rd country. In particular, I'm going to proceed the research focusing on the following aspects. First, I derive how each country's optimal subsidy (or tax) level varies as the number of firms that belong to each country (in the 3rd country) changes. I investigate the economic reason for the above result. Also, I look at how the change in

[4] This research is meaningful since the types of strategic trade policies become different whether the industry is closer to Cournot or Bertrand competition.

[5] Let's consider the case where a government supports subsidy to the firm belonging to its country in Bertrand competition. In this case, that firm's effective MC decreases and so it lowers the price, which leads to the price reduction of the competing firm. As a result, these aggressive strategies of the two firms have negative effect on the profit of each firm. Because each government anticipates this kind of result, each government chooses a tax instead of a subsidy.

the number of firms belonging to a country affects the social welfare of that country. Second, in the existing research, both governments' policy intervention reduces the net social welfare of each country for the case where there is only one firm belonging to each country. Here, I'd like to investigate how this result changes as the number of each country's firms changes. Third, in the earlier study, if only one government introduces policy (subsidy) intervention in the duopoly market, the profit of the competing country's firm has decreased. Here, we examine how the above result may change in the general oligopoly market where there are more firms in the 3rd country. Fourth, in the duopoly structure, it has been known that the reaction curve in the subsidy space has been downward sloping. In this research, I'd like to derive how the shape of the reaction curve is affected as the distribution of firms belonging to each country changes because the shape of the reaction curve would have an effect on the equilibrium level and the policy direction of each government.

The structure of this paper is as follows. In chapter 2, I introduce the model and derive the equilibrium for the case where both governments adopt strategic trade policies. Also, I conduct several comparative statics analyses, such as the relation between the subsidy change and welfare, the relation between the change in the number of firms and the equilibrium subsidy (or tax) level and the relation between the change in the distribution of firms and change in each country's welfare.

In chapter 3, we look at how the each country's welfare changes as both governments introduce policy, relative to the initial state where no government intervenes in the market. In particular, I examine how the economic effect of each country's policy intervention varies as the relative distribution of firms belonging to each country changes. In chapter 4, I summarize the main results and suggest some policy implications. Also, I point out the limitation of this paper and provide some direction for future research.

2 Basic model

Let's consider the situation where there is Cournot competition in the 3rd country market between two countries' (country 1 and country 2) firms. Suppose that there are k country 1's firms (firm 1, 2, ..., k) and (n-k) country 2's firms (firm k+1, ..., n) in this market. Assume that inverse market demand function is P=a- bQ (P is market price, Q is total output in the market, a, b > 0). Each firm has the same CRS (constant returns to scale) technology, and each firm's marginal (and average) cost is c. (c > 0). Each country's government could introduce the subsidy (or tax) for firms belonging to its country. For the subsidy case, country 1 (or 2) supports subsidy s_1 (or s_2) for the unit output. Now, we summarize some notations that would be used frequently in this paper as follows.

(Notation 1) (a) SW_k^0 indicates the social welfare of country k in the initial state's equilibrium (where no government intervenes in the economy.)

(b) $q_i(k,l)$, $\Pi_i(k,l)$ represents the equilibrium output and profit of firm i respectively, for the case where government 1 chooses k and government 2 chooses l (k=I or N, l=I or N. I indicates the government intervention and N indicates no intervention.) For example, $q_i(I,N)$, $\Pi_i(I,N)$ represent the equilibrium output and profit of firm i respectively, for the case where the government 1 intervenes and the government 2 does not intervene.

(c) $SW_1(k,l)$, $SW_2(k,l)$ represents social welfare of country 1 and 2 respectively, for the case where government 1 chooses k and government 2 chooses l. For example, $SW_1(N,I)$, $SW_2(N,I)$ represents social welfare of country 1 and 2 respectively, for the case where the government 1 does not intervene and the government 2 intervenes.

2.1 Equilibrium and comparative statics

I'd like to examine the case where the both governments introduce the intervention (subsidy or tax) policy. In this case, because each firm receives subsidy for unit output, the gross profit increases by the sub-

sidy. Therefore, the profit maximization problem of the country 1's firm (including subsidy) is as follows.

$$\Pi_i = \left(\mathrm{a} - \mathrm{bQ} - \mathrm{c} + s_1\right) q_i \quad \left(i = 1, 2, \ldots, k\right)$$

Next, the maximization problem of the country 2's firm is as follows.

$$\Pi_j = \left(\mathrm{a} - \mathrm{bQ} - \mathrm{c} + s_2\right) q_j \left(j = k+1,\ k+2,\ \ldots,\ n\right)$$

Now, from (FOC's) (first order conditions) of the above n maximization problems, equilibrium output and gross profit of each firm are as follows.

$$q_i = \frac{1}{b\left(n+1\right)}\left[m + \left(n-k+1\right)s_1 - \left(n-k\right)s_2\right]$$

$$q_j = \frac{1}{b\left(n+1\right)}\left[m - ks_1 + \left(k+1\right)s_2\right]$$

$$\Pi_i = \frac{1}{b\left(n+1\right)^2}\left[m + \left(n-k+1\right)s_1 - \left(n-k\right)s_2\right]^2$$

$$\Pi_j = \frac{1}{b\left(n+1\right)^2}\left[m - ks_1 + \left(k+1\right)s_2\right]^2 \ \left(\mathrm{i} = 1, 2, \ldots, \mathrm{k}\ ,\ \mathrm{j}=\mathrm{k}+1,\ \mathrm{k}+2,\ \ldots,\ \mathrm{n}\right)\ (\mathrm{m}=\mathrm{a}-\mathrm{c})$$

From the above result, I can check that output of country 1's firm is the increasing function of government 1's subsidy and the decreasing function of the government 2's subsidy. Namely, if country 1's subsidy increases, the perceived MC (marginal cost) of the country 1's firms decreases, which would lead to the output expansion of those firms. On the other hand, if country 2's subsidy increases, the effective MC of the country 2's firms goes down. And so the country 2's firms raise output, and the country 1's firms reduce outputs in response to this.

Now, we can derive the net profit (gross profit – subsidy) of each firm as follows.

$$\Pi_i^n = \Pi_i - s_1 q_i$$
$$= \frac{1}{b(n+1)^2}\left[m + (n-k+1)s_1 - (n-k)s_2\right]\left[m - ks_1 - (n-k)s_2\right]$$

$$\Pi_j^n = \Pi_j - s_2 q_2$$
$$= \frac{1}{b(n+1)^2}\left[m - ks_1 + (k+1)s_2\right]\left[m - ks_1 - (n-k)s_2\right]$$

$(\mathrm{i} = 1, 2, ..., \mathrm{k}\ \ \mathrm{j}=\mathrm{k}+1, \mathrm{k}+2, ..., \mathrm{n})$

Since the welfare of each country is the total net profits of the respective country's firms, we can derive it as follows.

$$SW_1 = k\Pi_i^n$$
$$= \frac{k}{b(n+1)^2}\left[m + (n-k+1)s_1 - (n-k)s_2\right]\left[m - ks_1 - (n-k)s_2\right]$$

$$SW_2 = (n-k)\Pi_j^n$$
$$= \frac{(n-k)}{b(n+1)^2}\left[m - ks_1 + (k+1)s_2\right]\left[m - ks_1 - (n-k)s_2\right]$$

Now, from the above result, we can derive the effect of a country's subsidy increase on the welfare of the other country as follows.

(Lemma 2-1) $\frac{\partial SW}{\partial}$, $\frac{\partial SW_2}{\partial s_1} < 0$ hold.

(Proof) The above results are derived easily from the social welfare expressions of the two countries. **Q.E.D.**

According to the above result, a country's welfare goes down if the competing country's subsidy increases. If country 2's subsidy rises, country 2's firms raise output. Therefore, the residual demand facing the country 1's firms goes down, which leads to the decrease in the output and profit of those firms. So country 1's welfare falls.

From the (FOC)'s of welfare maximization problem of each country,

we can derive the reaction function of each country as follows.

$$s_1 = R_1(s_2) = \frac{n(2k+1)m}{2k(n-k+1)} - \frac{(n-k)(n-2k+1)}{2k(n-k+1)} s_2$$

$$s_2 = R_2(s_1) = \frac{(2k+1-n)m}{2(k+1)(n-k)} - \frac{k(n-2k-1)}{2(k+1)(n-k)} s_1$$

From the intersection of the two reaction curves, equilibrium subsidies of both countries are as follows.

$$s_1(k,n) = \frac{(n-2k+1)m}{k(n+3)},\ s_2(k,n) = \frac{(2k+1-n)m}{(n-k)(n+3)}$$

We derive as follows the change in the subsidy of each country for the situation where the number of a country's firms increases, under the condition that the total number of firms in the market stays the same.

(Lemma 2-2) If the country 1's firms increase, then s_1 goes down and s_2 goes up.

(Proof) $\frac{\partial s_1}{\partial k} = -\frac{(n+1)m}{(n+3)k^2} < 0,\ \frac{\partial s_2}{\partial k} = -\frac{(n+1)m}{(n+3)(n-k)^2} > 0$ **Q.E.D**

For the above situation, if the number of country 1's firms increases and that of country 2's firms decreases, then country 1's subsidy falls and country 2's subsidy rises. That is, the subsidy of the country with more firms goes down, and that of the country with fewer firms goes up. We can explain the reason for this result as follows. If country 1 raises its subsidy, then each firm of the country 1 increases output. The subsidy policy of country 1 would have a negative profit effect on country 1's firms as well as country 2's firms.

Since the above negative effect on country 1's firms becomes larger as the number of country 1's firms rises, country 1 refrains from the expansionary subsidy policy, which results in a decrease in the subsidy. Meanwhile, as the number of country 1's firms decreases, the negative

influence of the subsidy policy of country 1 on country 1's firms becomes lower. So country 1 would adopt a more expansionary subsidy policy in this case.

Based on the above results, the relation between the number of each country's firms and the subsidy level is as follows.

(Lemma 2-3) If $k < \frac{n}{2}$, $s_1 > s_2$ holds.

(Proof) $s_1 - s_2 = \frac{(n-2k+1)}{k(n+3)}m - \frac{(2k+1-n)}{(n-k)(n+3)}m = \frac{(n-2k)(n+1)}{k(n-k)(n+3)}m > 0$ holds.

(if $k < \frac{n}{2}$) **Q.E.D**

According to the above result, country 1's subsidy is higher than country 2's if the number of country 1's firms is lower than that of country 2's. As the number of one country's firms decreases, that country would have incentive to adopt a more aggressive subsidy policy as explained above.

Now, from the above equilibrium subsidy level, the net profit of each firm and the welfare of each country are as follows.

$$\Pi_i^n = \frac{(n+1-k)m^2}{bk(n+3)^2},\ \Pi_j^n = \frac{(k+1)m^2}{b(n-k)(n+3)^2}$$

$$SW_1(I,I) = k\Pi_i^n = \frac{(n+1-k)m^2}{b(n+3)^2},\ SW_2(I,I) = (n-k)\Pi_j^n = \frac{(k+1)m^2}{b(n+3)^2}$$

Now, for the situation where the total number of firms stays the same, the effect of the change in the number of a country's firms on the net profit of each firm is as follows.

(Theorem 2-1) As the number of country 1's firms increases, the net profit of a country 1's firm decreases, and the net profit of a country 2's

firms increases.

(Proof) $\frac{\partial \Pi_i^n}{\partial k} = -\frac{(n+1)m^2}{bk^2(n+3)^2} < 0$ (i= 1, 2, ..., k)

$$\frac{\partial \Pi_j^n}{\partial k} = \frac{(n+1)m^2}{b(n-k)^2(n+3)^2} > 0 \quad (j = k+1, ..., n)$$

Q.E.D.

According to the above result, for the case where the total number of firms in the 3rd market always equals n, if the number of country 1's firms goes up and that of country 2's goes down, then the net profit of a country 1's firm decreases and that of country 2's rises. This result differs from the situation where no government introduces the strategic trade policy. Namely, for the case where there is no government intervention, the profit of each firm does not change when the number of one country's firms increases and that of the other country's decreases if the total number remains the same. But for the case like now where each country adopts the government policy, as the number of one country's firms decreases, the net profit of a firm belonging to that country increases. The country with a small number of firms adopts the more aggressive subsidy policy, which leads to the increase in the output and profit of that firm. In response to this, the output of the other country's firm decreases, which results in the decrease of the profit of that firm.

Now, the effect of a change in the number of firms belonging to a country on each country's welfare (for the situation where the total number of firms in the market does not change) is as follows.

(Theorem 2-2) As the number of country 1's firms increases, the welfare of country 1 falls and that of country 2 rises.

(Proof) $\frac{\partial SW_1(I,I)}{\partial k} = -\frac{m}{b(n+3)^2} < 0$, $\frac{\partial SW_2(I,I)}{\partial k} = \frac{m^2}{b(n+3)^2} < 0$ **Q.E.D**

The above result derives the change in the welfare of each country as the distribution of two country's firms varies, for the case where each government adopts the strategic trade policy. As the number of country 1's firms rises and that of country 2's falls, the welfare of country 1 decreases and that of country 2 increases. For this case, it would become difficult for country 1 to choose an aggressive subsidy policy because there are more country 1's firms that receive the negative profit effect by the subsidy increase. So country 1 chooses the more recessionary policy, and country 2 adopts the more expansionary policy. Therefore, the output and profit of the country 1's firms and the welfare of country 1 go down, while those of country 2's go up.

3 The economic effect of the policy intervention

In this chapter, I would like to investigate how the welfare of each country varies as the government of each nation introduces some policy. For the initial state where there is no policy intervention, the equilibrium and welfare of each country is as follows.[6]

(Lemma 3-1) (Initial State's Equilibrium)

$$\Pi_i^o = \frac{m^2}{b(n+1)^2} \quad (\text{i} = 1, 2, ..., \text{n})$$

$$SW_1^o = k\Pi_i^o = \frac{km^2}{b(n+1)^2},\ SW_2^o = (n-k)\Pi_j^o = \frac{(n-k)m^2}{b(n+1)^2}$$

Now, based upon the results derived until now, we would examine the equilibrium change by the introduction of each country's policy.

[6] The derivation is omitted because it is the standard Cournot competition case.

3.1 Symmetric case (The number of each country's firms is the same)

3.1.1 One government's intervention

Here, we consider the symmetric case where numbers of both country's firms are the same. Namely, the total number (n) is even and this is the case, $k=\frac{n}{2}$.

If we substitute $k=\frac{n}{2}$ to the reaction function of each country that is derived in the previous chapter, then the reaction function becomes as follows.

$$s_i = R_i\left(s_j\right) = \frac{m}{n\left(n+2\right)} - \frac{1}{\left(n+2\right)} s_j \quad (i=1,2,\ j=1,2)$$

Namely, the reaction curve of each country has a positive intercept and negative slope.[7]

Now, let's look at the situation where only government 1 introduces the policy and government 2 does not intervene in the policy. Since government 2 does not intervene in the policy, this case is the same as $s_2 = 0$. Then from the reaction function of country 1, we can derive $s_1 = \frac{2m}{n\left(n+2\right)}$ (>0).

Now, if we substitute country 1's equilibrium subsidy level into the profit and welfare formula, then the equilibrium levels are as follows.

(Lemma 3-2) (The Equilibrium where only government 1 intervenes)

$$q_i\left(I,N\right) = \frac{m}{bn},\ q_j\left(I,n\right) = \frac{m}{b\left(n+2\right)},\ \Pi_i\left(I,N\right) = \frac{m^2}{bn^2},\ \Pi_j\left(I,N\right) = \frac{m^2}{b\left(n+2\right)^2}$$

[7] The shape of reaction curve of the government i is determined by the strategic characteristics (strategic substitute or complement) of s_i chosen by that government. See the appendix for the detail.

$$\Pi_i^n(I,N)=\frac{m^2}{bn(n+2)},\quad SW_1=k\Pi_i^n=\frac{m^2}{2b(n+2)},\quad SW_2=(n-k)\Pi_j=\frac{nm^2}{2b(n+2)^2}$$

From the above result, we derive the change in the welfare of each country for the case where only country 1 adopts a strategic trade policy.

(Theorem 3-1) $SW_1(I,N)>SW_1^0,\ SW_2(I,N)<SW_2^0$

(Proof) $SW_1(I,N)-SW_1^o=\frac{m^2}{2b(n+2)}-\frac{nm^2}{2b(n+2)^2}=\frac{m^2}{b(n+2)^2}>0$

$$SW_2^o-SW_2(I,N)=\frac{nm^2}{2b(n+1)^2}-\frac{nm^2}{2b(n+2)^2}=\frac{n(2n+3)m^2}{2b(n+1)^2(n+2)^2}>0 \quad \textbf{Q.E.D}$$

From the above result, for the symmetric case, if only government 1 adopts a strategic policy, the welfare of country 1 goes up and that of country 2's goes down. In this case, country 1 chooses positive subsidy, and the country 1's firm raises output, and country 2's firm lowers output. So the profit of country 1's firm increases, and that of a country 2's decreases. Therefore, for the symmetric case, if only one government adopts a strategic policy, then the welfare of that country rises, while that of the other country is affected negatively.[8]

3.1.2 Both governments' intervention

Now, let's examine the case where both governments adopt strategic policies. From the 2 reaction functions of the two countries, the equilibrium subsidy is $s_1=s_2=\frac{m}{n(n+3)}$ for this case. Also, we compare the welfare of each country for this case with that of the initial state as follows.

(Theorem 3-2) $SW_1(I,I)\left(=SW_2(I,I)\right)<SW_1^0\left(=SW_2^0\right)$

[8] Because of this kind of characteristic, both the governments are involved in the policy intervention, as analyzed before.

(Proof) $SW_i^0 - SW_i(I,I) = \frac{nm^2}{2b(n+1)^2} - \frac{(n+1)m^2}{2b(n+3)^2}$

$$= \frac{3n(n+2)m^2}{2b(n+1)^2(n+3)^2} > 0 \qquad \textbf{Q.E.D}$$

According to the above result, for the symmetric case, if both countries adopt intervention policies, then the net welfare of each country becomes lower than that of the initial state. In this case, both countries choose the positive subsidy in a policy game. By this result, output of each firm rises and the market prices falls compared to the initial state. So the net profit of each firm and the net welfare of each country decrease. This situation is similar to the 'prisoner's dilemma' because the policy intervention of the two countries results in the welfare decrease of both countries.

3.2 Asymmetric case (Numbers of each country's firms are different)

In this section, I'm going to examine the asymmetric market structure case where the numbers of each country's firms are not the same. For this analysis, I divide the cases into two; one case is that the difference of numbers is 1 and the other case is that the difference is more than 1.

3.2.1 Case where difference of firms' numbers is one

3.2.1.1 One government intervention

Let's consider the case where the number of country 1's firms is higher than that of country 2's by one. This is the case where total number n is odd and $k = \frac{1}{2}(n+1)$. If we substitute $k = \frac{1}{2}(n+1)$ in each country's reaction function derived in the previous chapter, the reaction functions of both governments are derived as follows.

$$s_1 = R_1(s_2) = 0$$

$$s_2 = R_2(s_1) = \frac{4m}{(n+3)(n-1)} - \frac{2(n+1)}{(n+3)(n-1)} s_1$$

That is, the reaction curve of government 1 is a vertical line through the origin, and that of government 2 is a straight line with a negative slope and a positive intercept.

(1) Government 1's intervention

Here we examine the case where only government 1 adopts an intervention policy. This case is the same as the case where the government 2 always set s_2 as 0. Also, from the two reaction curves derived above, we know $s_1 = 0$. Therefore, since s_1 and s_2 are set 0 in this equilibrium, this case is the same as no intervention. Therefore, welfares of each country in this case are the same as those of the initial state (no government intervention case).

(2) Government 2's intervention

Here we look at the case where only government 2 chooses intervention. This case is the same as the case where the government 1 always set $s_1 = 0$. So if we substitute $s_1 = 0$ into the reaction curve of government 2, then the government 2's optimal subsidy is $s_2 = \frac{4m}{(n+3)(n-1)}$. That is, in this case government 2 adopts the positive subsidy, which has the role of inducing output expansion of country 2's firms. By substitution of this subsidy into welfare functions, we derive the welfare of both countries as follows.

$$SW_1(\mathrm{N,I}) = \frac{(n+1)m^2}{2b(n+3)^2},\ SW_2(\mathrm{N,I}) = \frac{(n-1)m^2}{2b(n+1)^2}$$

Now, we compare the welfare of this case with that of the initial state as follows.

(Theorem 3-3) $SW_1(N,I) < SW_1^0$, $SW_2(N,I) > SW_2^0$

(Proof) $$SW_1^0 - SW_1(N,I) = \frac{m^2}{2b(n+1)} - \frac{(n+1)m^2}{2b(n+3)^2} = \frac{2(n+2)m}{b(n+1)(n+3)^2} > 0$$

$$SW_2(N,I) - SW_2^0 = \frac{m^2}{2b(n+3)} - \frac{(n-1)m^2}{2b(n+1)^2} = \frac{2m^2}{b(n+3)(n+1)^2} > 0 \qquad \textbf{Q.E.D}$$

For the situation where only government 2 intervenes, country 1's welfare becomes lower and country 2's welfare becomes higher than that of the initial state. In this case, government 2 adopts the expansionary policy, which leads to the increase in output and profit of country 1's firms and the decrease in those of country 2's firms. Accordingly, the welfare of country 1 rises and that of country 2 falls.

3.2.1.2 Both governments' intervention

Now we look at the case with both governments' intervention. From reaction functions derived above, the reaction of government 1 is always $s_1 = 0$. So in this case, from the reaction function of country 2, the optimal subsidy of country 2 is $s_2 = \frac{4m}{(n+3)(n-1)}$. That is, in the equilibrium in the policy game, government 1 chooses 0 subsidy and government 2 adopts the positive subsidy. In this case, the reaction curve of country 1 is the vertical line through the origin, and that of country 2 slopes downward, which leads to the equilibrium as above. We may explain the above results as follows. If outputs of a country's firms increase by the expansionary subsidy policy, then it would exert negative profit effect on its own country's firms as well as those of the competing country. This is the situation where country 1 has more firms than country 2. So the positive subsidy policy of country 1 would have more negative effect on country 1's firms than country 2's firms. Therefore, government 1 would refrain from the expansionary subsidy policy and it adopts 0 subsidy in the equilibrium. Also, because country 2 has less firms than country 1, it adopts expansionary positive subsidy in the equilibrium.

Now, because the above equilibrium is the same as the case where only government 2 intervenes, the following lemma holds.

(Lemma 3-3) If $k = \frac{1}{2}(n+1)$, $SW_1(I,I) = SW_1(N,I)$, $SW_2(I,I) = SW_2(N,I)$

(Proof) From the previous argument, s_1, s_2 are the same in the two cases so that welfares of both countries in the two cases are the same. **Q.E.D**

So welfares for the case of both government's intervention are the same as those for the case where only government 2 intervenes. Namely in this case, the welfare of country 1 becomes lower and that of country 2 becomes higher compared to the initial state.

3.2.2 Case where difference of firms' numbers is more than one

Here we consider the case where the number of country 1's firms is higher than that of country 2's by more than one. This case corresponds to the case where $k > \frac{1}{2}(n+1)$ in the previous model.

3.2.2.1 One government intervention

(1) Government 1's intervention

Let's consider the case where only government 1 intervenes in the above situation. This case is equal to the case where government 1 always sets $s_2 = 0$. So if we substitute $s_2 = 0$ into the government 1's reaction function, then we get $s_1 = R_1(s_2 = 0) = \frac{(n+1-2k)m}{2k(n-k+1)}$ (<0). Namely, in this case, the government 2's optimal policy is not to support subsidy, but to exercise tax to its firms.

We can derive welfares of both countries by replacing the above equilibrium value to welfare equations of both countries as follows.

$$SW_1(I,N)=\frac{m^2}{4b(n-k+1)},\ SW_2=\frac{(n-k)m^2}{4b(n-k+1)^2}$$

We compare welfares of this case with those of the initial state as follows.

(Theorem 3-4) $SW_1(I,N)>SW_1^0,\ SW_2(I,N)>SW_2^0$

(Proof) $SW_1(I,N)-SW_1=\frac{m^2}{4b(n-k+1)}-\frac{km^2}{b(n+1)^2}=\frac{(2k-n-1)\ m}{4b(n-k+1)(n+1)^2}>0$

$$SW_2(I,N)-SW_2^0=\frac{(n-k)m^2}{4b(n-k+1)^2}-\frac{(n-k)m^2}{b(n+1)^2}$$

$$=\frac{(n-k)m^2}{4b(n-k+1)^2(n+1)^2}(3n+3-2k)(2k-n-1)>0$$

(since $3n+3-2k>0$ by $k\le n$ and $2k-n-1>0$ by $k>\frac{1}{2}(n+1)$). **Q.E.D**

According to the above result, welfares of both countries go up if only government 1 intervenes. The reasons for this result are as follows. For this case where only government 1 intervenes, the government 1 adopts tax rather than subsidy. As we have examined above, if the number of a country's firms increases, then that country chooses a contractionary policy, so the subsidy level decreases in the new equilibrium. So for the case like now where country 1's firm number is greater than that of country 2 by more than 1, the equilibrium subsidy level decreases until it becomes negative value. Now if government 1 exercises tax to its own firms, then country 1's firms reduce output. So the total output in the market decreases and the market price increases, which has a positive profit effect on country 2's firms. Therefore, welfares of both countries increase.

(2) Government 2's intervention

Now let's examine the situation where only the government 2 in-

tervenes. This corresponds to the case where government 1 always sets $s_1 = 0$. So if we substitute s_1 as 0 to reaction functions derived above, then we get the optimal $s_2 = \frac{2k+1-n}{2(k+1)(n-k)}$ (> 0). Now by substituting this equilibrium subsidy to welfare equations of both countries, we get welfares of both countries as below.

$$SW_1(N,I) = \frac{km^2}{4b(k+1)^2},\ SW_2(N,I) = \frac{m^2}{4b(k+1)}$$

Now, we compare welfares of both countries in this case with those of the initial state as follows.

(Theorem 3-5) $SW_1(N,I) < SW_1^0,\ SW_2(N,I) > SW_2^0$ holds.

(Proof) $SW_1^0 - SW_1(N,I) = \frac{km^2}{b(n+1)^2} - \frac{km^2}{4b(k+1)^2}$

$$= \frac{km^2}{4b(n+1)^2(k+1)^2}(2k+n+3)(2k+1-n) < 0 \text{ holds,}$$

by the assumption $k > \frac{n+1}{2}$.

$$SW_2(N,I) - SW_2^0 = \frac{m^2}{4b(k+1)} - \frac{(n-k)m^2}{b(n+1)^2} = \frac{(2k+1-n)^2}{4b(k+1)(n+1)^2} > 0 \text{ holds,}$$

by the assumption $k > \frac{n+1}{2}$. **Q.E.D.**

From the above result, the welfare of country 2 goes up and that of country 1 goes down if only government 2 intervenes in this case. As in the case like now where the number of country 2's firms are smaller than that of country 1's by more than one, government 2 adopts the expansionary subsidy policy. As a result, the output and profit of a country 2's firm increase and those of a country 1's firm decrease. So the welfare of the former rises and that of the latter falls.

3.2.2.2 Both governments' intervention

Now let's look at the situation where both governments intervene. From the reaction functions derived in the previous chapter, shapes of reaction curves are as follows. Namely, the reaction curve of a country 1 is a straight line with a negative intercept and a positive slope. The reaction curve of a country 2 is a straight line with a positive intercept and a negative slope. Also, in this situation the equilibrium values of s's are

$$s_1 = \frac{(n-2k+1)m}{k(n+3)} < 0 \,,\; s_2 = \frac{(2k+1-n)m}{(n-k)(n+3)} > 0$$

According to the above result, government 1 adopts tax (or negative subsidy) and government 2 chooses subsidy in the equilibrium. This is the situation where the number of country 1's firms are sufficiently larger than that of country 2's. So if government 1 chooses expansionary positive subsidy in this case, then there occurs a large negative profit effect on country 1's firms. By this reason, government 1 chooses tax on its firms, which induces the output reduction of country 1's firms, which in turn lowers the total output and so raises the market price.

Now by using equilibrium subsidies, we derive welfares of both countries as below.

$$SW_1(I,I) = \frac{(n+1-k)m^2}{b(n+3)^2} \,,\; SW_2(I,I) = \frac{(k+1)m^2}{b(n+3)^2}$$

We're going to compare this result with the case where only one government intervenes.

(1) Comparison with the case of government 1's intervention

We compare welfares of both countries in this case with those where only government 1 intervenes as follows.

(Theorem 3-6)

(a) $SW_1(I,N) > SW_1^0 > SW_1(I,I)$ holds.

(b) $SW_2(I,I) > SW_2(I,N) > SW_2^0$ holds.

(Proof) (a) $SW_1(I,N) - SW_1^0 = \frac{m^2}{4b(n-k+1)} - \frac{km^2}{b(n+1)^2} = \frac{(2k-n-1)^2 m^2}{4b(n-k+1)(n+1)^2} > 0$

$$SW_1^0 - SW_1(I,N) = \frac{km^2}{b(n+1)^2} - \frac{(n+1-k)m^2}{b(n+3)^2} = \frac{m^2}{b(n+1)^2(n+3)^2} f(n,k)$$

$$\text{(where, } f(n,k) = (2k-n-1)n^2 + 2(4k-n-1)n + 10k - n - 1)$$

Now it is sufficient to show $f(n,k) > 0$. Since we have $k > \frac{n+1}{2}$ by the assumption, $2k-n-1>0$, $4k-n-1>0$, $10k-n-1>0$ hold, resulting in $f(n,k) > 0$.

(b-1) $SW_2(I,N)$ is country 2's welfare for the case where only government 1 intervenes. By the previous result, if government 2 intervenes additionally, then country 2's optimal subsidy has been positive. From the stand-point of government 2, country 2 could get the welfare that is the same as the case where government 2 does not intervene by choosing s_2=0. So the fact that government 2 adopts the subsidy other than 0 implies that country 2's welfare must increase by government 2's additional intervention. So the relation $SW_2(I,I) > SW_2(I,N)$ holds.

(b-2) $SW_2(I,N) - SW_2^0 = \frac{(n-k)m^2}{4b(n-k+1)^2} - \frac{(n-k)m^2}{b(n+1)^2}$

$$= \frac{(n-k)m^2}{4b(n-k+1)^2(m+1)^2} g(n,k)$$

$$\text{(where, } g(n,k) = (3n-2k+3)(2k-n-1))$$

It suffices to show $g(n,k) > 0$. Now since $k < n$ and $3n-2k+3>0$ and since $2k-n-1>0$ by the assumption $k > \frac{n+1}{2}$, $g(n,k) > 0$ holds. **Q.E.D.**

By the above result, from the situation where a government 1 intervenes at first, the additional intervention of a government 2 leads to the increase of the country 2's welfare and decrease of the country 1's welfare. As in the above, for the case where only government 1 intervenes, welfares of both countries go up because government 1 chooses tax (instead of subsidy) in this case. From this situation, if a government 2 intervenes in addition, then the government 2 adopts a positive subsidy. So the output and profit of a country 2's firm rise and those of a country 2's falls. Therefore the country 2's welfare becomes higher and country 1's welfare becomes lower relative to the case where only the government 1 intervenes.

(2) Comparison with the case of government 2's intervention

Now we compare welfares of both countries in this case with those where only government 2 intervenes.

(Theorem 3-7) (a) $SW_1^0 > SW_1(I,I) > SW_1(N,I)$ holds.

(b) $SW_2(I,I) > SW_2(N,I) > SW_2^0$ holds.

(Proof)

(a-1) $SW_1^0 - SW_1(I,I) = \dfrac{m^2}{2b(n+1)} - \dfrac{(n+1)m^2}{2b(n+3)^2} = \dfrac{2(n+2)m}{b(n+1)(n+3)^2} > 0$ holds.

(a-2) $SW_1(N,I)$ is the country 1's welfare for the case where only government 2 intervenes. By the previous result, if the government 1 intervenes additionally, then country 1's optimal subsidy has been negative. From the stand-point of government 1, country 1 could get the welfare that is the same as the case where government 1 does not intervene by choosing $s_1 = 0$. So the fact that government 1 adopts the subsidy (or tax) other than 0 implies that country 1's welfare must increase by government 1's additional intervention. So the relation $SW_1(I,I) > SW_1(N,I)$ holds.

(b-1) According to the above result,

$$SW_2(I,I) - SW_2(N,I) = \frac{(k+1)m^2}{b(n+3)^2} - \frac{m^2}{4b(k+1)} = \frac{(2k-n-1)(2k+n+5)}{4b(k+1)(n+3)^2} > 0 \text{ holds,}$$

since $k > \frac{n+1}{2}$ by the above assumption.

(b-2) $SW_2(N, I) > SW_2^0$ has been proved before. **Q.E.D.**

By the above result, from the situation where a government 2 intervenes at first, the additional intervention of a government 1 leads to the increase in the welfare of country 2 as well as that of country 1. From this situation, if a government 1 intervenes in addition, the government 1 chooses tax whereas the government 2 adopts subsidy. So the perceived marginal cost of country 1's firms increases, which results in the output reduction of those firms. As a result, the residual demand facing country 2's firms increases, which leads to the increase in the profit of country 2's firms[9] and so has a positive welfare effect on the country 2.

So the additional intervention of government 1 for the case where a government 2 intervenes at first has a positive welfare effect on country 2. Namely, for the situation where the country with fewer firms intervenes in the first place, the additional intervention of the country with more firms, leads to the increase in the welfare of the country with fewer firms. Therefore, a government's intervention does not necessarily have a negative influence on the welfare of the other country. We need to note that the welfare effect of a government's intervention on the other countries relies on the distribution of firms belonging to each country.

4 Summary and conclusion

In this paper, I examine the welfare effect of strategic trade policy of each country for the case where each country's firms enter the 3rd country. Namely, I analyze the situation where each government introduces subsidy (or tax) policy. I investigate how the policy and welfare of each country vary as the distribution of the number of firms that belong to

[9] The decrease of firms' outputs belonging to the country 1 leads to the decrease of the total market output, which results in the increase of the market price. And the market price increase has positive profit effects on country 2's firms.

each country changes. Next, I look at how the welfare of each country alters as both governments adopt intervention policies, compared to the initial state where there is no government intervention.

Now, the main results of this paper are as follows. First, if the number of firms that belong to country 1 increases, country 1's subsidy decreases and country 2's subsidy goes up. In the related result, if the number of country 1's firms is lower than that of country 2's, country 1's subsidy is higher than that of country 2's. Second, if the number of country 1's firms rises, the net profit of a country 1's firm and country 1's welfare falls. On the other hand, the net profit of a country 2's firm and country 2's welfare increases. Third, if country 1's subsidy goes up, the net profit of a country 2's firm and country 2's welfare goes down. Fourth, if the number of firms that belong to each country is equal, each government adopts a (positive) subsidy in equilibrium. In this equilibrium, each firm's output increases and the net profit and each country's welfare are lower than those of the initial state with no government intervention. Also, in this case each country's subsidy becomes a strategic substitute because the reaction curves of each government slopes downward in the subsidy-space.[10] Fifth, let's consider the case where the number of country 1's firms is one more than that of country 2's. In this case, government 1 adopts no subsidy and government 2 adopts positive subsidy in equilibrium. Namely, in the subsidy-space, the reaction curve of government 1 is a vertical line through the origin, and that of government 2 is downward sloping. From the standpoint of government 1, to adopt a positive subsidy and induce output expansion of country 1's firms would have a negative effect on the country 1's firms as well as country 2's firms. Now since the country 1's firms are one more that country 2's in this case, government 1 refrains from this kind of expansionary policy and so the optimal subsidy becomes 0.[11] Meanwhile, if we compare the new equilibrium with the initial state, the country 1's welfare decreases and country 2's welfare increases.

[10] This situation is similar to the case of prisoner's dilemma. Namely, whereas the dominant strategy of each government is the intervention, each country's social welfare becomes higher for the case where no government intervenes.

[11] The government 2's expansionary subsidy has negative profit effects on country 1's firms. But the government 2 adopts the positive subsidy since it does not take into account this kind of negative external effect.

Sixth, let's consider the situation where the number of country 1's firms is higher than country 2's by more than 1. In this case, the government 1 adopts tax and government 2 adopts subsidy in the equilibrium. For the case where the number of country 1's firms are sufficiently high relatively, the negative profit effect on country 1's firms becomes very large if government 1 adopts the (expansionary) positive subsidy. Therefore, the government would like to raise market power by inducing price increase through output reduction of country 1's firms by imposing a tax. In response to the tax imposition by the government 1, the government 2 adopts a subsidy, which has negative influence on country 1's firms. Therefore, the welfare of country 1 goes down and that of country 2 rises in the new equilibrium compared to the initial state.

Here, one interesting result is that government 1's policy intervention exerts a positive external effect on country 2's welfare, regardless of whether the government 2 intervenes or not.[12] Seventh, for the asymmetric market structure case where numbers of each country's firms differ, the policy intervention equilibrium does not lie in the prisoner's dilemma situation. Namely, in this asymmetric structure case, one country's (here, country 2) welfare increases relative to the initial state in the policy equilibrium.

Now, we summarize the policy implications of this research as follows. First, for the symmetric structure case where numbers of each country's firms are equal, welfares of both countries decrease in the new policy equilibrium relative to the initial state. So in this case there may be some cooperation incentive for both governments not to intervene in the policy. Second, for the asymmetric structure case where there are differences in the number of each country's firms, the welfare of one country (here, country 2) increases in the policy equilibrium. So government 2 would have the incentive to move into the new equilibrium by triggering the policy intervention game.

Now, the limitation of this paper and the future research directions are as follows. First, in this paper, I analyze the case of linear demand

[12] In this case, the government 1's reaction curve is upward sloping, and the government 2's reaction curve is downward sloping. Namely, in policy introduction, the government 2 chooses aggressively, while the government 1 behaves softly (or less aggressively).

and cost. Later, it would be better to look at the general demand and cost case. If we examine the general demand and cost case, then we could figure out how much the research results are sensitive to assumptions. Second, here I look at the Cournot competition case with homogeneous goods. Later, it would be necessary to extend the analysis to the Bertrand competition case with differentiated products. Third, in this research I investigate the situation where two countries' firms enter the 3rd country market. Next, if we look at the case where there are domestic and foreign firms in a country, then we could find out the difference between the competition in the domestic market and that in the 3rd country market. Fourth, in this paper, I examine the situation where the number of firms competing in the 3rd country market is given. Later, we could comprehend how these results are robust to the model assumption if we deal with the free-entry case.

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Appendix

Here I would like to examine how the strategic characteristics (strategic substitute or strategic complement) of subsidy (or tax) is determined according to the difference in the relative distribution of numbers of firms belonging to each country.

(Lemma A-1)

(a) If $k > \frac{n+1}{2}$, s_1 becomes a strategic complement and the government 1's reaction curve is upward sloping. In contrast, s_2 becomes a strategic substitute and the government 2's reaction curve is downward sloping.

(b) If $k = \frac{n+1}{2}$, s_1 is strategically neutral and the government 1's reaction curve is a vertical line through the origin. While, s_2 becomes a strategic substitute and the government 2's reaction curve is downward sloping.

(c) If $k = \frac{n}{2}$, s_1 and s_2 become strategic substitutes, and both governments' reaction curves are downward sloping

(Proof)

(a) If $k > \frac{1}{2}(n+1)$, $\frac{\partial SW_1^2}{\partial s_1 \partial s_2} = -\frac{k}{b(n+1)^2}(n-k)(n-2k+1) > 0$ and $\frac{\partial SW_2^2}{\partial s_1 \partial s_2} = -\frac{k}{b(n+1)^2}(n-2k-1) < 0$ holds.

(b) If $k = \frac{1}{2}(n+1)$, $\frac{\partial SW_1^2}{\partial s_1 \partial s_2} = -\frac{k}{b(n+1)^2}(n-k)(n-2k+1) = 0$ and $\frac{\partial SW_2^2}{\partial s_1 \partial s_2} = -\frac{k}{b(n+1)^2}(n-2k-1) < 0$ holds.

(c) If $k = \frac{n}{2}$, $\frac{\partial SW_1^2}{\partial s_1 \partial s_2} = -\frac{k}{b(n+1)^2}(n-k)(n-2k+1) < 0$ and

$\frac{\partial SW_2^2}{\partial s_1 \partial s_2} = -\frac{k}{b(n+1)^2}(n-2k-1) < 0$ holds. **Q.E.D.**