Final-offer arbitration and successive elimination of weakly dominated offers*

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Abstract

We analyze the final-offer salary arbitration when the parties have perfect information on the evaluation of the arbitrator. We use a discrete model in which the parties offer discrete numbers. We apply the criterion of successive elimination of weakly dominated offers to identify the final outcomes. We found that attitudes toward risk affected outcomes. When parties are risk-averse, the outcome of the arbitration is equal to the arbitrator's evaluation. When the parties are risk-neutral or weakly risk-loving, the outcome diverges slightly from the arbitrator's evaluation. When parties are strongly risk-loving, the outcomes are dispersed over a wide range of offers.

Keywords: Final-Offer Arbitration, Weakly Dominated Offers, Discrete Model, Attitudes toward Risk

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1 Introduction

In final-offer arbitration, the two parties in a dispute submit final offers to an arbitrator. The arbitrator then chooses one of the two offers. Similar procedure was used to determine Socrates' punishment after he was convicted in Athens, B.C. 399. The origination of the concept of final-offer arbitration (FOA), however, is credited to Stevens (1966) and the FOA has since been used to arbitrate many disputes. For instance, it has been used for salary arbitration in Major League Baseball since 1973. Many states in the United States use the FOA to settle disputes with public employees in essential public services, such as firefighting, nursing, and police services.

Strikes as a last resort in labor-management disputes are banned for public security purposes in the case of employees of essential public services. Instead, employees were granted the right to resort to arbitration. Conventional arbitration is coercive. However, coercive arbitration has several limitations. Parties may complain about the arbitration results because of the arbitrator's arbitrary decisions. To avoid this, the arbitrator tends to compromise and split the pie between the claims of both parties, which induces them to present excessive claims to their advantage. The arbitrator may also 'split the baby' while compromising the two parties' claims (Chetwynd 2009). In addition, the arbitrator has the heavy burden of producing a reasonable compromise. This arbitration procedure required considerable time and hard work. To overcome these difficulties, the FOA was introduced. Under this procedure, the burden of evidence collection and related data presentation is distributed to the parties, which eases the burden on the arbitrator and decentralizes the process, thereby reducing complaints regarding the arbitrariness of arbitration. The FOA is more conducive to settlement than conventional arbitration because it exposes the parties to the risk of losing the case and being swayed by the other party's offer. It is also more conducive to compromise and considerate offers than conventional arbitration because a compromised offer raises the probability of adoption by the arbitrator.

The final-offer salary arbitration of the MLB illustrates the successful operation of the FOA. According to Hill and Jolly (2014), the FOA is conducted for each player with a service time of three to six years. Either eligible players or owners can file for arbitration between January 5 and 15, following a season. Players and clubs exchange offers on January 18. The league schedules arbitration hearings between February 1 and 20. A three-member arbitration panel usually renders a decision within 24 hours of the conclusion of the hearing.

Existing literature examines the Nash equilibrium of FOA games. For instance, Crawford (1979) shows that there is a unique Nash equilibrium in an FOA with perfect information. Faber (1980), Chatterjee (1981), and Brams and Merrill (1983) consider the Nash equilibrium of the FOA with imperfect information on the player's proper value in the mind of the arbitrator.

We use the successive elimination of weakly dominated offers (strategies) as a criterion for sorting the solution of the FOA game. We consider a FOA with perfect information on the value of a player. We identify pairs of offers (strategy profiles) that survive the successive elimination of weakly dominated offers (SEWDO) under various attitudes toward the risks of the parties. The analysis shows that as parties become risk-loving, the outcome tends to diverge from the arbitrator's evaluation.

The remainder of this paper is organized as follows: Section 2 presents the basic model and an example. Section 3 presents the main results of the FOA under perfect information when the parties are risk-averse. Section 4 presents the results of the FOA when the parties are risk-neutral. Section 5 presents the results of the FOA when the parties are risk-loving. Finally, Section 6 concludes the paper.

2 A Model of Final Offer Arbitration and An Example

There is a club (or buyer) and a player (or seller). There is an arbitrator who thinks that the appropriate amount of the player's salary is $v > 0$. Both the club and player know the amount v of salary in the mind of the arbitrator. The club offers an amount x of salary, and the player offers an amount y of salary. The value of v takes a discrete value $1, 2, \ldots$. The values of x and y also take discrete values $0, 1, 2, \ldots$. Then, an arbitrator chooses the one closest to the amount v between the two offers by the club and player. If the two offers are at the same distance from v, the arbitrator chooses each offer with a probability of 1/2.

The FOA is modeled as a game form in which the club and player are participants, and the strategies of the club and player are salary offers. The outcome function $g(x, y)$ of the game form denotes the player's arbitraged value (salary) when the pair of salary offers is (x, y). When $x + y = 2v, x \neq y$, the outcome (salary) is probabilistic. The probabilistic outcome is represented by a distribution function F over non-negative real numbers. We call it a lottery l. Agent j has a preference relation R^j over lotteries, $j = c, p$ where c is for club and p is for player. The preference relation is complete, transitive, continuous, and satisfies the independence axiom. When $l\ R^j\ l'$ and not $l' R^j\ l$ for two lotteries l, l', we say that j strictly prefers l to l' and denote that $l\ P^j\ l'$. When $l\ R^j\ l'$ and $l'\ R^j\ l$, we say that j is indifferent between l and l' and denote that $l\ I^j\ l'$. The preference over the lotteries varies according to attitudes toward risk. We introduce a certainty-equivalent outcome function $g^c(x, y)$ for club and $g^p(x, y)$ for player. The certainty equivalent of a lottery l for agent j is the monetary outcome (salary) z such that $1_z \sim^j l$ where 1_z is the lottery such that the probability of z is 1. The game form can be represented by a table in which the numbers in the leftmost cells denote the club's offer and the numbers in the uppermost cells indicate the player's offer. The numbers in the other cells denote the arbitraged salaries of the player. Note that, when the outcome is probabilistic, both $g^c(x, y)$ and $g^p(x, y)$ are inscribed in the cell. The club prefers a lower salary, and the player prefers a higher salary.

Player is risk-averse if the certainty equivalent of a lottery is less than the expected value of the lottery and larger than the minimum amount with positive probability of the lottery. Player is extremely risk-averse if the certainty equivalent of a lottery is equal to the minimum amount of the lottery. Player is risk-neutral if the certainty equivalent of a lottery is equal to the expected value of the lottery. Player is risk-loving if the certainty equivalent of a lottery is larger than the expected value of the lottery and less than the maximum amount of the lottery. Player is extremely risk-loving if the certainty equivalent of a lottery is equal to the maximum amount of the lottery. Symmetric definition applies to club's attitude towards risk. Later, we will define the weak risk-lovingness and the strong risk-lovingness when we prove the related results.

Now, suppose that $v = 3$, and that the club and player are both risk-averse. The game form of the FOA is presented in Table 1.

An offer (or strategy) x for the club is *weakly dominated* by another offer

x' if x results in an arbitraged salary that is larger than or equal to that under another offer x' regardless of the player's offer, and larger than that for at least one offer chosen by the player. The weakly dominated offer for the player is defined similarly. We denote these weak dominance relations for club and player as $x \prec^c x'$ and $y \prec^p y'$ respectively. An offer is *undominated* if it is not weakly dominated by any other offers.

In the example where $v = 3$ and both the club and player are risk-averse ($3 < g^c(x,y) < \max(x,y), \min(x,y) < g^p(x,y) < 3$ when $x + y = 2v, x \neq y$), we obtain a unique solution $(x,y) = (3,3)$, applying the successive elimination of weakly dominated offers. The successive elimination procedure is as follows:

Table 1. The normal form of final-offer arbitration where the arbitrator's evaluation $v = 3$

Club \ Player	0	1	2	3	4	5	6	7	8	9	…
0	0	1	2	3	4	5	g^p<3 <g^c	0	0	0	
1	1	1	2	3	4	g^p<3 <g^c	1	1	1	1	
2	2	2	2	3	g^p<3 <g^c	2	2	2	2	2	
3	3	3	3	3	3	3	3	3	3	3	
4	4	4	g^p<3 <g^c	3	4	4	4	4	4	4	
5	5	g^p<3 <g^c	2	3	4	5	5	5	5	5	
6	g^p<3 <g^c	1	2	3	4	5	6	6	6	6	
7	0	1	2	3	4	5	6	7	7	7	
8	0	1	2	3	4	5	6	7	8	8	
9	0	1	2	3	4	5	6	7	8	9	
⋮											

First round of elimination of weakly dominated offers

We identified the weakly dominated offers of club and player. Note that if the offer y of player is weakly dominated by another offer y' of player, then the offer $x' = y'$ of the club is weakly dominated by the offer $x = y$, and vice versa.

The club's weakly dominated offers are as follows:

The club's offer $x = 6,7, \ldots$ is weakly dominated by offer $x' = 0$. The club's offer $x = 5$ is weakly dominated by offer $x' = 1$. Generally, a club's offer $x = v + i, i = 1,2,3$ is weakly dominated by the offer $x' = v - i$.

Thus, the club's offers that survive the elimination of weakly dominated offers are $x = 0,1,2,3$. This is because the club's offer $x = 0,1,2,3$ is the best response to the player's offer $y = 7 - x$.

The player's weakly dominated offers are as follows:

Player's offer $y = 7$ is weakly dominated by another offer $y' = 8$. The player's offer $y = 8$ is weakly dominated by $y' = 9$. Generally, the player's offer $y = 2v + i, i = 1,2, \ldots$ is weakly dominated by $y' = 2v + i + 1$.

The player's offer $y = 0$ is weakly dominated by $y' = 6,7,8, \ldots$.

The player's offer $y = 1$ is weakly dominated by $y' = 5$. Generally, the player's offer $y = v - i, i = 1,2,3$ is weakly dominated by the offer $y' = v + i$, as shown in Table 1.

Thus, the offers that survive the elimination of weakly dominated offers are $y = 3,4,5,6$. This is because the player's offer $y = 3,4,5$ is the best response to the club's offer $x = 5 - y$. The player's offer $y = 6$ is better than the player's offer $y' = 0,1,2,3,4,5$ when club's offer $x = 6$ and is better than the player's offer $y' = 7,8,9, \ldots$ when club's offer $x = 0$.

Second round of elimination of weakly dominated offers

Table 2. Second round of elimination of weakly dominated offers

		Player			
		3	4	5	6
Club	0	3	4	5	g^p<3<g^c
	1	3	4	g^p<3<g^c	1
	2	3	g^p<3<g^c	2	2
	3	3	3	3	3

Club's offer 0 is weakly dominated by offer 3.

Player's offer 6 is weakly dominated by offer 3.

Third round of elimination of weakly dominated offers

Table 3. Third round of elimination of weakly dominated offers

		Player		
		3	4	5
Club	1	3	4	g^p<3<g^c
	2	3	g^p<3<g^c	2
	3	3	3	3

Club's offer 1 is weakly dominated by offer 3.
Player's offer 5 is weakly dominated by offer 3.
Similarly, in the fourth round of elimination, the club's offer 2 is weakly dominated by offer 3, and the player's offer 4 is weakly dominated by offer 3.
After eliminating the weakly dominated offers, we obtain the pair of offers $(x, y) = (3,3)$.

3 The Case Where the Parties are Risk-averse

The outcomes of the arbitration can be classified into four cases, with each case corresponding to the area in Table 4.

Case 1: $x < y, x + y < 2v \rightarrow |x - v| > |y - v|$ (player wins), $g^c(x, y) = g^p(x, y) = y$

Case 2: $x < y, x + y > 2v \rightarrow |x - v| < |y - v|$ (club wins), $g^c(x, y) = g^p(x, y) = x$

Case 3: $x > y, x + y > 2v \rightarrow |x - v| > |y - v|$ (player wins), $g^c(x, y) = g^p(x, y) = y$

Case 4: $x > y, x + y < 2v \rightarrow |x - v| < |y - v|$ (club wins), $g^c(x, y) = g^p(x, y) = x$

If $x \neq y$ and $x + y = 2v$, then $v < g^c(x, y) < \max(x, y)$ and $\min(x, y) < g^p(x, y) < v$, because the parties are risk-averse. If $x = y$, then $g^c(x, y) = g^p(x, y) = x = y$.

Table 4. The normal form of final-offer arbitration in the first round

player

		0	1	…	v-i	…	v-1	v	v+1	…	v+i	…	2v-1	2v	2v+1	2v+2	…
	0	0	1	…	v-i	⋯	v-1	v	v+1	…	v+i	⋯	2v-1		0	0	…
	1	1	1	…	v-i	⋯	v-1	v	v+1	…	v+i	⋯		1	1	1	…
	⋮	⋮	⋮		⋮		⋮	⋮	⋮		⋮		⋮	⋮	⋮	⋮	
	v-i	v-i	v-i	…	v-i	⋯	v-1	v	v+1	…		⋯	v-i	v-i	v-i	v-i	…
	⋮	⋮	⋮		⋮		⋮	⋮	⋮		⋮		⋮	⋮	⋮	⋮	
	v-1	v-1	v-1	…	v-1	⋯	v-1	v		…	v-1	⋯	v-1	v-1	v-1	v-1	…
club	v	v	v	…	v	⋯	v	v	v	…	v	⋯	v	v	v	v	…
	v+1	v+1	v+1	…	v+1	⋯		v	v+1	…	v+1	⋯	v+1	v+1	v+1	v+1	…
	⋮	⋮	⋮		⋮		⋮	⋮	⋮		⋮		⋮	⋮	⋮	⋮	
	v+i	v+i	v+i			⋯	v-1	v	v+1		v+i	⋯	v+i	v+i	v+i	v+i	⋯
	⋮	⋮	⋮		⋮		⋮	⋮	⋮		⋮		⋮	⋮	⋮	⋮	
	2v-1	2v-1		…	v-i	⋯	v-1	v	v+1	…	v+i	⋯	2v-1	2v-1	2v-1	2v-1	…
	2v		1	…	v-i	⋯	v-1	v	v+1	…	v+i	⋯	2v-1	2v	2v	2v	…
	2v+1	0	1	…	v-i	⋯	v-1	v	v+1	…	v+i	⋯	2v-1	2v	2v+1	2v+1	…
	2v+2	0	1	…	v-i	⋯	v-1	v	v+1	…	v+i	⋯	2v-1	2v	2v+1	2v+2	…
	⋮	⋮	⋮		⋮		⋮	⋮	⋮		⋮		⋮	⋮	⋮	⋮	

Theorem 1: When parties are risk-averse, the successive elimination of weakly dominated offers (SEWDO) ends after $v+1$ rounds of elimination. Profile $(x,y)=(v,v)$ survives SEWDO.

Proof:

Step 1: The case of $v=1$

The outcome table is as follows:

Table 5. The outcome table when the parties are risk-averse

Player

Club		0	1	2	3	4	⋯
	0	0	1		0	0	⋯
	1	1	1	1	1	1	⋯
	2		1	2	2	2	⋯
	3	0	1	2	3	3	⋯
	4	0	1	2	3	4	⋯
	⋮	⋮	⋮	⋮	⋮	⋮	

When $x+y=2, x \neq y, 1 < g^c(x,y) < \max(x,y)$.

Note that $2 \prec^c 0$. Also note that $2+i \prec^c 0, i=1,2,\cdots$.

Club's offer 0 is undominated because it is the best response to player's offer 3. (When $1 \leq x \leq 3$, $g^c(0,3)=0 < g^c(x,3)=x$ and when $x>3$, $g^c(0,3)=0 < g^c(x,3)=3$.)

Club's offer 1 is undominated because it is the best response to player's offer 2. (Note that $g^c(1,2)=1 < g^c(0,2)$ and when $x \geq 2, g^c(1,2)=1 < g^c(x,2)=2$.)

As for the player, note that $\min(x,y) < g^p(x,y) < 1$ when $x+y=2, x \neq y$.

We can easily show that $2+i \prec^p 2+i+1, i=1,2,\cdots$ and $0 \prec^p 2$.

And player's offer 1 is undominated because it is the best response to the club's offer of zero.

Player's offer 2 is undominated because it is better than 3,4,5,... against club's offer 0, and it is better than 0,1 against club's offer 2.

After the first round of elimination, club's offer 0 is weakly dominated by the offer of 1: $0 \prec^c 1$. And player's offer 2 is weakly dominated by the offer of 1: $2 \prec^p 1$.

After the second round of elimination, there remains only the offer profile $(x,y)=(1,1)$

Step 2: The case of $v > 1$

The outcome table is as follows:

Table 6. The outcome table when the parties are risk-averse

Player

Club	0	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v$	$2v+1$	$\cdots$
0	0	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$		0	$\cdots$
1	1	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$		1	1	$\cdots$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$v+1$		$\vdots$	$\vdots$	$\vdots$	
$v-1$	$v-1$	$v-1$	$\cdots$	$v-1$	v		$\cdots$	$v-1$	$v-1$	$v-1$	$\cdots$
v	v	v	$\cdots$	v	v	v	$\cdots$	v	v	v	$\cdots$
$v+1$	$v+1$	$v+1$	$\cdots$		v	$v+1$	$\cdots$	$v+1$	$v+1$	$v+1$	$\cdots$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$	
$2v-1$	$2v-1$		$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v-1$	$2v-1$	$\cdots$
$2v$		1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v$	$2v$	$\cdots$
$2v+1$	0	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v$	$2v+1$	$\cdots$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$	

We can show that $2v \prec^c 0$. Note that the following inequalities hold:

For $y = 0,\ g^c(0,y) = 0 \leq v < g^c(2v,y)$.

For $0 < y < 2v,\ g^c(0,y) = y = g^c(2v,y)$.

For $y = 2v,\ g^c(0,y) < \max(0,y) = y = g^c(2v,y)$.

For $y > 2v, g^c(0,y) = 0 < 2v = g^c(2v,y)$.

We can show that $2v + i \prec^c 0, i = 1,2,\cdots$. Note that the following inequalities hold:

For $y < 2v,\ g^c(0,y) = 0 = g^c(2v+i,y)$.

For $y = 2v,\ g^c(0,y) < \max(0,y) = y = g^c(2v+i,y)$.

For $2v < y \leq 2v+i,\ \ g^c(0,y) = 0 < y = g^c(2v+i,y)$.

For $y > 2v+i,\ g^c(0,y) = 0 < 2v+i = g^c(2v+i,y)$.

We can show that $v+i \prec^c v-i, i=1,2,\cdots,v-1$. Note that the following inequalities hold:

For $y < v-i, g^c(v-i,y) = v-i < v+i = g^c(v+i,y)$.

For $y = v-i, g^c(v-i,y) = v-i < v < g^c(v+i,y)$.

For $y = v, g^c(v-i,y) = y = g^c(v+i,y)$.

For $y = v+i, g^c(v-i,y) < \max(v-i,y) = v+i = g^c(v+i,y)$.

For $y > v+i, g^c(v-i,y) = v-i < v+i = g^c(v+i,y)$.

The club's offer $i=0$ is undominated because it is the best response to the player's offer $2v+1$.

For $0 < x \leq 2v+1$, $g^c(x,2v+1) = x > 0 = g^c(0,2v+1)$.

For $x > 2v+1$, $g^c(x,2v+1) = 2v+1 > 0 = g^c(0,2v+1)$.

Club's offer $i=1,\cdots,v$ is undominated because it is the best response to player's offer $2v+1-i$.

For $x < i-1, g^c(x,2v+1-i) = 2v+1-i > i = g^c(i,2v+1-i)$.

For $x = i-1, g^c(x,2v+1-i) > v \geq i = g^c(i,2v+1-i)$.

For $i < x \leq 2v+1-i, g^c(x,2v+1-i) = x > i = g^c(i,2v+1-i)$.

For $x > 2v+1-i, g^c(x,2v+1-i) = 2v+1-i > i = g^c(i,2v+1-i)$.

Now we show that $2v+i \prec^p 2v+i+1, i=1,2,\cdots$.

For $x \leq 2v+i, g^p(x,2v+i) = x = g^p(x,2v+i+1)$.

For $x > 2v+i, g^p(x,2v+i) = 2v+i < 2v+i+1 = g^p(x,2v+i+1)$.

We show that $v-i \prec^p v+i, i=1,2,\cdots,v-1$.

For $x < v-i, g^p(x,v-i) = v-i < v+i = g^p(x,v+i)$.

For $x = v-i, g^p(x,v-i) = v-i = \min(x,v+i) < g^p(x,v+i)$.

For $x = v, g^p(x,v-i) = x = g^p(x,v+i)$.

For $x = v+i, g^p(x,v-i) < v < v+i = g^p(x,v+i)$.

For $x > v+i, g^p(x,v-i) = v-i < v+i = g^p(x,v+i)$.

We show that $0 \prec^p 2v$.

For $x = 0, g^p(x,2v) > \min(x,2v) = 0 = g^p(x,0)$.

For $0 < x \leq 2v-1, g^p(x,2v) = x = g^p(x,0)$.

For $x = 2v, g^p(x,2v) = 2v > v > g^p(x,0)$.

For $x > 2v, g^p(x,2v) = 2v > 0 = g^p(x,0)$.

Player's offer $v+j, j=0,1,\cdots,v-1$ is undominated because it is the best response to club's offer $v-1-j$.

For $y \le v-1-j, g^p(v-1-j,y) = v-1-j < v+j = g^p(v-1-j,v+j)$

For $v-1-j < y \le v+j, g^p(v-1-j,y) = y < v+j = g^p(v-1-j,v+j)$

For $y = v+j+1, g^p(v-1-j,y) < v \le v+j = g^p(v-1-j,v+j)$

For $y > v+j+1, g^p(v-1-j,y) = v-1-j < v+j = g^p(v-1-j,v+j)$

Player's offer $2v$ is undominated because it is better than $2v+j, j = 1,2,\dots$ against club's offer 0 (that is, $g^p(0,y) = 0 < g^p(0,2v)$ for $y \ge 2v+j$) and it is better than 0 against club's offer $2v$ ($g^p(2v,0) < v < 2v = g^p(2v,2v)$), and it is better than $y = 1,\dots,2v-1$ against club's offer $2v$ ($g^p(2v,y) = y < 2v = g^p(2v,2v)$).

After round $n = 1$ elimination, the outcome table is as follows:

Table 7. The outcome table when the parties are risk-averse

Club \ Player	v	$v+1$	$\cdots$	$2v-1$	$2v$
0	v	$v+1$	$\cdots$	$2v-1$	
1	v	$v+1$	$\cdots$		1
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$v-1$	v		$\cdots$	$v-1$	$v-1$
v	v	v	$\cdots$	v	v

We show that $0 <^c v$.

For y such that $v \le y \le 2v-1$, it holds that $g^c(0,y) = y \ge v = g^c(v,y)$.

For $y = 2v$, it holds that $g^c(0,y) > v = g^c(v,y)$.

Club's offer $x = i,\ i = 1,2,\cdots,v$ is undominated. This is because $x = i$ is the best response to the player's offer $2v+1-i$.

Now we show that $2v \prec^p v$.

For $x = 0$, it holds that $g^p(x, v) = v > g^p(x, 2v)$.

For x such that $0 < x \leq v - 1$, it holds that $g^p(x, v) = v > g^p(x, 2v) = x$.

For $x = v$, it holds that $g^p(x, v) = v = g^p(x, 2v)$.

We can show that player's offer $v + i, i = 0,1, \cdots, v - 1$ is undominated. It is because $y = v + i, i = 0,1, \cdots, v - 1$ is the best response to club's offer $x = v - 1 - i$.

After round $n \geq 1$ elimination, the outcome table is as follows:

Table 8. The outcome table when the parties are risk-averse

Club \ Player	v	$v+1$	$\cdots$	$2v-n$	$2v+1-n$
$n-1$	v	$v+1$	$\cdots$	$2v-n$	
n	v	$v+1$	$\cdots$		n
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$v-1$	v		$\cdots$	$v-1$	$v-1$
v	v	v	$\cdots$	v	v

We show that $n - 1 \prec^c v$.

For y such that $v \leq y \leq 2v - n$, it holds that $g^c(n - 1, y) = y \geq v = g^c(v, y)$.

For $y = 2v + 1 - n$, it holds that $g^c(n - 1, y) > v = g^c(v, y)$.

Club's offer $x = i, i = n, n + 1, \cdots, v$ is undominated because it is the best response to player's offer $2v + 1 - i$.

We show that $2v + 1 - n \prec^p v$.

For $x = n - 1$, it holds that $g^p(x, 2v + 1 - n) < v = g^p(x, v)$.

For x such that $n \leq x \leq v - 1$, it holds that $g^p(x, 2v + 1 - n) = x \leq v = g^p(x, v)$.

Player's offer $v + j, j = 0, \cdots, v - n$ is undominated because it is the best response to club's offer $v - 1 - j$.

After round $n = v + 1$, the outcome table is as follows: And the surviving outcome is the offer profile $(x, y) = (v, v)$.

Table 9. The outcome table after $n = v + 1$ rounds when the parties are risk-averse

		Player
		v
Club	v	v

Q.E.D.

In the case where the parties are extremely risk-averse (i.e., if $x + y = 2v$, then $g^c(x, y) = \max(x, y)$, $g^p(x, y) = min(x, y)$), we can show the following result: (For the proof, refer to the Appendix).

Theorem 2: When the parties are extremely risk-averse,

a) If v is odd, the profiles of the offers surviving SEWDO are the offer profiles $(x, y) = (v - 1, v), (v, v)$.

b) If v is even, the profiles of offers that survive SEWDO are offer profiles $(x, y) = (v, v), (v, v + 1)$.

4 The Case Where the Parties are Risk-neutral

In this case, both the club and the player face the same outcome table. Because $g^c(x, y) = g^p(x, y) = v$, when $x + y = 2v$, we can let $g(x, y) = g^c(x, y) = g^p(x, y)$ for all x and y without affecting the solution of the FOA.

Theorem 3: When the parties are risk-neutral, the profile of offers for the surviving SEWDO procedure is $(x, y) = (v - 1, v + 1)$.

Proof:

For the case where $v = 1, 2$, the theorem is easily verified.

For the case where $v \geq 3$, we show that the following holds in a manner similar to the proof of Theorem 1:

1. $0 \succ^c 2v - 1 + i, i = 0,1,2,\cdots$
2. $v - 1 \succ^c v$
3. $v - i - 1 \succ^c v + i, i = 1,2,\cdots,v - 1$
4. Club's offer $i, i = 0, 1, 2, \cdots v - 1$ is undominated because it is the best response to player's offer $2v + 1 - i$.

Thus, the club's offer $i, i = 0, 1, \ldots, v - 1$ survives the first round of eliminating weakly dominated offers.

For player, similar results hold.

1. $2v + i \prec^p 2v + i + 1, i = 1,2,\cdots$
2. $2v - i \succ^p i, i = 0,1,\cdots v - 1$
3. $v + 1 \succ^p v$
4. Player's offer $v + i, i = 1,2,\cdots,v - 1$ is undominated because it is the best response to club's offer $v - 1 - i$.
5. Player's offer $2v$ is undominated because it is better than $2v + i, i = 1,2,\cdots$ for club's offer 0, and better than $i, i = 0,\cdots,2v - 1$ for club's offer $2v$.

Thus, the player's offer $i, i = v + 1, \ldots, 2v$ survives the first round of eliminating weakly dominated offers.

After the first round of elimination, we obtain the following outcome table:

Table 10. The outcome table after the first round of elimination when both parties are risk-neutral

		Player			
		$v+1$	$\cdots$	$2v-1$	$2v$
Club	0	$v+1$	$\cdots$	$2v-1$	v
	1	$v+1$	$\cdots$	v	1
	$\vdots$	$\vdots$		$\vdots$	$\vdots$
	$v-1$	v	$\cdots$	$v-1$	$v-1$

For the club, the following holds in the second round of elimination:

1. $0 \prec^c v-1$
2. $i, i = 1, \cdots, v-1$ is undominated because it is the best response to player's offer $2v+1-i$.

For the player, the following holds in the second round of elimination:

1. $2v \prec^p v+1$
2. $v+i, i = 1, \cdots, v-1$ is undominated because it is the best response to club's offer $v-1-i$.

After $n, n = 1,2, \cdots v$ rounds of elimination, the outcome table is as follows:

Table 11. The outcome table after the $n, n = 1,2, \cdots v$ round of elimination when both parties are risk-neutral

Player (columns); Club (rows)

	$v+1$	$\cdots$	$2v-n$	$2v-n+1$
$n-1$	$v+1$	$\cdots$	$2v-n$	v
n	$v+1$	$\cdots$	v	n
$\vdots$	$\vdots$		$\vdots$	$\vdots$
$v-1$	v	$\cdots$	$v-1$	$v-1$

For club, the following holds:

1. $n-1 \prec^c v-1$
2. Club's offer $i, i = n, \cdots v-1$ is undominated. This is because it is the best response to player's offer $2v+1-i$.

For player, the following holds:

1. $2v-n+1 \prec^p v+1$
2. Player's offer $v+i, i = 1, \cdots, v-n$ is undominated. This is because this is the best response to the club's offer $v-1-i$.

After v rounds of elimination, only the offer profile $(x, y) = (v-1, v+1)$ remains. In other words, it survived the SEWDO. Q.E.D.

5 The Case Where the Parties Are Risk-loving

When $x + y = 2v\ and\ x \neq y$, it holds that $\min(x, y) < g^c(x, y) < v$. For clubs, the following dominance relationship holds:

1. $v + i \prec^c v - i, i = 1,2, \cdots, v - 1$
2. $2v \prec^c 0$
3. $2v + i \prec^c 0, i = 1,2, \cdots$
4. $v \prec^c v - 1$
5. The club's offer $i = 0$ is undominated because it is the best response to the player's offer $2v + 1$.
   Club's offer $i, i = 1,2, \cdots, v - 2$, is undominated because it is better than $i + 1, i + 2, \cdots$ for player's offer $2v + 1 - i$ and it is better than $0,1, \cdots i - 1$ for player's offer $2v - i$.
   The club's offer $i = v - 1$ is undominated because it is the best response to the player's offer. $v + 1$.

Table 12. The outcome table when both parties are risk-loving

Player

Club

	0	⋯	v	$v+1$	$v+2$	⋯	$2v-1$	$2v$	$2v+1$	⋯
0		⋯	v	$v+1$	$v+2$	⋯	$2v-1$		0	⋯
1		⋯	v	$v+1$	$v+2$	⋯		1	1	⋯
⋮			⋮	⋮	⋮		⋮	⋮	⋮	
$v-2$				$v+1$			$v-2$	$v-2$	$v-2$	
$v-1$		⋯	v		$v-1$	⋯	$v-1$	$v-1$	$v-1$	⋯
v		⋯	v	v	v	⋯	v	v	v	⋯
$v+1$		⋯	v	$v+1$	$v+1$	⋯	$v+1$	$v+1$	$v+1$	⋯
⋮			⋮	⋮	⋮		⋮	⋮	⋮	
$2v-1$		⋯	v	$v+1$	$v+2$	⋯	$2v-1$	$2v-1$	$2v-1$	⋯
$2v$		⋯	v	$v+1$	$v+2$	⋯	$2v-1$	$2v$	$2v$	⋯
$2v+1$		⋯	v	$v+1$	$v+2$	⋯	$2v-1$	$2v$	$2v+1$	⋯
⋮			⋮	⋮	⋮		⋮	⋮	⋮	

For player, the following dominance relation holds: Note that, when $x + y = 2v\ and\ x \neq y$, it holds that $v < g^p(x, y) < \max(x, y)$.

1. $v + i \succ^p v - i, i = 1, \cdots, v - 1$
2. $2v \succ^p 0$
3. $v + 1 \succ^p v$
4. $2v + i + 1 \succ^p 2v + i, i = 1,2, \cdots$
5. The player's offer $v + 1$ is undominated because it is the best response to the club's offer $v - 1$.
   Player's offer $v + i, i = 2, \cdots, v - 1$ is undominated because it is better than $y = 0,1, \cdots, v + i - 1$ for club's offer $v - 1 - i$ ($g^p(v - 1 - i, y) < g^p(v - 1 - i, v + i)$) and it is better than $y > v + i$ for club's offer $v - i$ ($g^p(v - i, y) < g^p(v - i, v + i)$)
6. Player's offer $2v$ is undominated because it is better than $2v + i, i = 1,2, \cdots$ for club's offer 0, and it is better than $i, i = 0, \cdots, 2v - 1$ for club's offer $2v$

After first round of elimination, the outcome table is as follows:

Table 13. The outcome table after the first round of elimination when both parties are risk-loving ($v \geq 3$)

	$v+1$	$\cdots$	$2v-1$	$2v$
0	$v+1$	$\cdots$	$2v-1$	
1	$v+1$	$\cdots$		1
$\vdots$	$\vdots$		$\vdots$	$\vdots$
$v-2$	$v+1$	$\cdots$	$v-2$	$v-2$
$v-1$		$\cdots$	$v-1$	$v-1$

We say that the club is weakly risk-loving if $\min(x, y) + 1 \leq g^c(x, y) < v$ where $x + y = 2v, \min(x, y) \neq v - 1, v$. Players are weakly risk-loving if $v < g^p(x, y) \leq \max(x, y) - 1$ where $x + y = 2v, \max(x, y) \neq v + 1, v$. In the next theorem, we show that the unique surviving offer profile is $(x, y) = (v - 1, v + 1)$ when they are weakly risk-loving.

Theorem 4: When the parties are weakly risk-loving, the unique offer profile of the surviving SEWDO is $(x, y) = (v - 1, v + 1)$.

Proof:

When $v = 1, 2$, the theorem is easily verified. Thus suppose $v \geq 3$.

In the second round of elimination, we can show that $0 \prec^c 1$.

And the club's offer $i, i = 1, \cdots, v-1$ is undominated by the same reason as in the first round of elimination.

We can show that $2v \prec^p 2v-1$.

The player's offer $v+i, i = 1, \cdots, v-1$ is undominated by the same reason as in the first round of elimination.

After the second round of elimination, the outcome table is as follows:

Table 14. The outcome table after second round of elimination when both parties are risk-loving $(v \geq 3)$

	$v+1$	$v+2$	$\cdots$	$2v-2$	$2v-1$
1	$v+1$	$v+2$	$\cdots$	$2v-2$	
2	$v+1$	$v+2$	$\cdots$		2
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$v-2$	$v+1$		$\cdots$	$v-2$	$v-2$
$v-1$		$v-1$	$\cdots$	$v-1$	$v-1$

In the third round, we can show that $1 \prec^c 2$.

Club's offer $i, i = 2, \cdots, v-1$ is undominated.

We can show that $2v-1 \prec^p 2v-2$

Player's offer $v+i, i = 1, \cdots, v-2$ is undominated.

After kth round of elimination, $1 \leq k \leq v-1$, the outcome table is as follows:

Table 15. The outcome table after kth round of elimination $(v \geq 3)$

	$v+1$	$\cdots$	$2v$	$\cdots$	$k+1$
$k-1$	$v+1$	$\cdots$	$k-1$	$\cdots$	$k+1$
$\vdots$	$\vdots$		$\vdots$		$\vdots$
$v-1$		$\cdots$	$v-1$	$\cdots$	$v-1$

After $v-1^{\text{th}}$ round of elimination, the outcome table is as follows:

Table 16. The outcome table after $v-1$th round of elimination $(v \geq 3)$

	$v+1$	$v+2$
$v-2$	$v+1$	
$v-1$		$v-1$

We can show that $v-2 <^c v-1$ and $v+2 <^p v+1$.

After v^{th} round of elimination, the outcome table is as follows:

Table 17. The outcome table after v^{th} round of elimination Q.E.D.

	$v+1$
$v-1$	

We say that the club and player are strongly risk-loving, if they are risk-loving but not weakly risk-loving. If the parties are strongly risk-loving, then there is some integer $k(\neq v, v-1)$ such that $g^c(k, 2v-k) < k+1$ and $g^p(k, 2v-k) > 2v-k-1, 0 \leq k \leq v-2$. Let m be the smallest one among such numbers ($k(\neq v, v-1)$s).

Theorem 5: When the parties are strongly risk-loving, then all the offers $m, \cdots, v-1$ $(v \geq 2)$ by the club and all the offers $v+1, \cdots, 2v-m$ by the player are undominated where m is defined as above.

Proof:

After the first round of elimination, club offers $0, \ldots, v-1$ and player offers $v+1, \ldots, 2v$ survive. If $m = 0$, the outcome table is as shown in Table 18. If $m \geq 1$, in each round of elimination from round 2 to $m+1$ round, the same logic applies as in Theorem 4. Thus, club offers, $0, \ldots, m-1$ are eliminated one by one, and player offers, $2v, 2v-1, \ldots, 2v-m+1$ are eliminated one by one. After $m+1$ round of elimination, the outcome table is as follows:

Table 18. The outcome table after $m+1$ round of elimination

	$v+1$	$v+2$	$\cdots$	$2v-m-1$	$2v-m$
m	$v+1$	$v+2$	$\cdots$	$2v-m-1$	
$m+1$	$v+1$	$v+2$	$\cdots$		$m+1$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$v-2$	$v+1$		$\cdots$	$v-2$	$v-2$
$v-1$		$v-1$	$\cdots$	$v-1$	$v-1$

In the table above, the club's offer m is undominated because it is the best response to the player's offer $2v-m$. The club's offer $m+i$, $i = 1, \ldots, v-m-2$ is undominated because it is better than $m, \ldots, m+i-1$ for player's offer $2v-m-i$ and better than $m+i+1, \ldots, v-1$ for player's offer $2v-m-i+1$. The club's offer $v-1$ is undominated because it is the best response to the player's offer $v+1$.

The player's offer $2v-m$ is undominated because it is the best response to the club's offer m. The player's offer $2v-m-i$, $i = 1, \ldots, v-m-2$ is undominated because it is better than $v+1, \ldots, 2v-m-i-1$ for club's offer $m+i-1$ and better than $2v-m-i+1, \ldots, 2v-m$ for club's offer $m+i$. The player's offer $v+1$ is undominated because it is the best response to the club's offer $v-1$.

Q.E.D.

This theorem shows that when the parties are strongly risk-loving, the surviving offers from the SEWDO are a block of undominated offers.

In the case where the parties are extremely risk-loving (that is, $g^c(x,y) = \min(x,y)$, $g^p(x,y) = \max(x,y)$ when $x+y=2v$), note that the club's offer $v-i$, $i=1,\cdots v$ is undominated because it is the best response to the player's offer $v+i$. The player's offer $v+i, i=1,\cdots,v$ is undominated because it is the best response to club's offer $v-i$. Thus, the surviving offers from the SEWDO are a block of offers ranging from 0 to $v-1$ for the club and the surviving offers are a block of offers ranging from $v+1$ to $2v$ for player, in accordance with the theorem 5 where $m=0$.

Table 19. Surviving offers from SEWDO under various attitudes toward risk

Attitude Toward Risk	Surviving Offers From the SEWDO
Extremely Risk-Averse	v is odd: $(v,v),(v-1,v)$ v is even: $(v,v),(v,v+1)$
Risk-Averse	(v,v)
Risk-Neutral	$(v-1,v+1)$
Weakly Risk-Loving	$(v-1,v+1)$
Strongly Risk-Loving	Club: m, $\cdots, v-1$ Player: $v+1, v+2, \cdots, 2v-m$

6 Concluding Remarks

We analyze the FOA when parties know the evaluation of the proper salary by the arbitrator. We use a discrete model in which parties offer a discrete number as a salary offer. We use the successive elimination of weakly dominated offers as the solution concept. The solution outcome varies according to the parties' attitudes toward risk. When the parties are risk-averse, the pair of offers that survives the SEWDO is $(x,y)=(v,v)$. When the parties are risk-neutral, the pair of offers surviving the SEWDO is

$(x, y) = (v-1, v+1)$. When the parties are weakly risk-loving, the pair of offers that survives the SEWDO is $(x, y) = (v-1, v+1)$. When the parties are strongly risk-loving, there remains a block of offers that survive the SEWDO for clubs and players.

In the discrete model, the outcome of FOA changes as attitudes toward risk vary. The change is subtle. This change may be more significant when the grid size is large. In the continuous model, where the grid size approaches zero, both the surviving offer profiles and Nash equilibria coincide with $(x, y) = (v, v)$ when the parties are risk-averse or risk-neutral.

Note also that if we change the tie-breaking rule such that the average offers are chosen as the final salary when there is a tie, then the outcome is not random and risk attitude becomes irrelevant in determining the outcome. For instance, suppose that both the arbitrator and the parties submit sealed envelopes containing their salary offer, and the final salary is the average offer of the two parties when the two offers are equally distant from the evaluation of the arbitrator. In this modified FOA game, the outcome is the same as that in the unmodified FOA, in which the parties are risk-neutral.

Our paper contributes to the literature in three ways. First, we set up a discrete model while the existing literature uses continuous models. Discrete model is of reality while continuous models are approximations to the reality in the FOA. For instance, club or player does not offer the salary $\sqrt{2}$ dollars in the real world. Under the discrete model, we can obtain the divergent outcome in FOA in the context of perfect information on the arbitrator's evaluation. In the context of imperfect information, the divergent outcome is obtained as a general result (Brams and Merill III (1983)). Thus, the discrete model is better in that it shows the divergent outcome consistently under both the perfect information and the imperfect information.

Second, the Nash equilibrium implicitly presupposes the existence of a coordination mechanism such as pre-play communication. This supposition is necessary for the self-confirming expectation property of Nash equilibrium. The supposition of pre-play communication is why the Nash equilibrium is often interpreted as a self-enforcing agreement. Thus, if we do not suppose the existence of a coordination mechanism, Nash equilibrium may not be convincing. For instance, in the chicken game where each player may choose evasion or confrontation, it is natural not to suppose the pre-play communication. In this case, both players may choose confrontations. The successive elimination of weakly dominated strategies does not presuppose the existence of coordination mechanisms. Thus, the solution outcome obtained using the solution concept SEWDO is more robust than that obtained by the Nash equilibrium.

Third, our analysis includes the case where the parties are risk-loving while the existing literature deals only with the cases where the parties are risk-averse or risk-neutral.

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Appendix: The proof of theorem 2

Theorem 2: When the parties are extremely risk-averse,

(a) If v is odd, offer profiles $(x,y) = (v-1, v), (v, v)$ survive the SEWDO procedure.

(b) If v is even, then the offer profiles $(x,y) = (v,v), (v, v+1)$ survive the SEWDO procedure.

Proof:

Case 1: $v = 1$

The outcome table is as follows:

Table A1. The outcome table when the parties are extremely risk-averse $(v = 1)$

		Player				
		0	1	2	3	⋯
Club	0	0	1	0	0	⋯
	1	1	1	1	1	⋯
	2	0	1	2	2	⋯
	3	0	1	2	3	⋯
	⋮	⋮	⋮	⋮	⋮	

The following dominance relations hold for club:

Club's offers $2,3,4\cdots$ are weakly dominated by offer 0: $2,3,4\cdots \prec^c 0$

Club's offers 0,1 are undominated because 0 is the best response to player's offer 3, and 1 is the best response to player's offer 2.

The following dominance relation holds for player:

$2 \prec^p 3 \prec^p 4 \prec^p \cdots$

$0 \prec^p 1$

Player's offer 1 is undominated because 1 is the best response to the club's offer of 0.

Case 2: $v = 2$
The outcome table is as follows:

Table A2. The outcome table of club when the parties are extremely risk-averse $(v = 2)$

Player

Club	0	1	2	3	4	5	6	⋯
0	0	1	2	3	4	(0)	0	⋯
1	1	1	2	3	(1)	1	1	⋯
2	2	2	2	(2)	2	2	2	⋯
3	3	3	2	3	3	3	3	⋯
4	4	1	2	3	4	4	4	⋯
5	0	1	2	3	4	5	5	⋯
6	0	1	2	3	4	5	6	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	

The following dominance relation holds for club:
$4,5,6 \cdots \prec^{c} 0$
$3 \prec^{c} 1$
Club's offers 0,1,2 are undominated because club offer 0 is the best response to player offer 5, 1 is the best response to player offer 4, and 2 is the best response to player offer 3.
The outcome table facing player is as follows:

Table A3. The outcome table of player when the parties are extremely risk-averse

Player

Club	0	1	2	3	4	5	6	⋯
0	0	1	2	(3)	0	0	0	⋯
1	1	1	(2)	1	1	1	1	⋯
2	2	2	2	2	2	2	2	⋯

3	3	1	2	3	3	3	3	⋯
4	0	1	2	3	4	4	4	⋯
5	0	1	2	3	4	5	5	⋯
6	0	1	2	3	4	5	6	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	

The following dominance relation holds for player:

$4 \prec^p 5 \prec^p 6 \prec^p \cdots$

$0 \prec^p 3$

$1 \prec^p 2$

Player's offers 2,3 are undominated because player's offer 2 is the best response to club's offer 1 and player's offer 3 is the best response to club's offer 0.

After first round of elimination, the outcome table facing club is as follows:

Table A4. The outcome table of club after first round of elimination when the parties are extremely risk-averse

Club \ Player	2	3
0	2	3
1	2	3
2	2	(2)

The following dominance relation holds for club:

$0,1 \prec^c 2$

A club's offer 2 is undominated because it is the best response to player's offer 3.

The outcome table facing player is as follows:

Table A5. The outcome table of player after first round of elimination when the parties are extremely risk-averse

Club \ Player	2	3
0	2	(3)
1	(2)	1
2	2	2

The following dominance relation holds for player:

Player's offers 2,3 are undominated because player's offer 2 is the best response to club's offer 1 and player's offer 3 is the best response to club's offer 0.

After the second round of elimination, the outcome table for both clubs and players is as follows:

Table A6. The outcome table of both club and player after second round of elimination when the parties are extremely risk-averse

Club \ Player	2	3
2	2	2

In general, let $v = 2k - 1$ or $2k$, where $k \geq 2$ and $n = 2l - 1, l = 1,2,\cdots,k$.

Now let $n = 1$.

In the first round ($n = 1$) of elimination, the outcome table for the club is as follows:

Table A7. The outcome table of club when the parties are extremely risk-averse

Player

		0	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v$	$2v+1$	$\cdots$
	0	0	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v$	0	$\cdots$
	1	1	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	1	1	$\cdots$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$v+1$		$\vdots$	$\vdots$	$\vdots$	
Club	$v-1$	$v-1$	$v-1$	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$v-1$	$v-1$	$v-1$	$\cdots$
	v	v	v	$\cdots$	v	v	v	$\cdots$	v	v	v	$\cdots$
	$v+1$	$v+1$	$v+1$	$\cdots$	$v+1$	v	$v+1$	$\cdots$	$v+1$	$v+1$	$v+1$	$\cdots$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$	
	$2v-1$	$2v-1$	$2v-1$	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v-1$	$2v-1$	$\cdots$
	$2v$	$2v$	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v$	$2v$	$\cdots$
	$2v+1$	0	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v$	$2v+1$	$\cdots$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$	

The following dominance relation holds for club:

1. $2v+1, 2v+2 \cdots \prec^c 0 \rightarrow 2v+i \prec^c 0,\ i = 1,2,\cdots$
2. $v+i \prec^c v-i,\ i = 1,2,\cdots,v$
3. $i, i = 0,\cdots,v$ is undominated since it is the best response to player's offers $2v+1-i$.

The outcome table facing player is as follows:

Table A8. The outcome table of player when the parties are extremely risk-averse

Club \ Player	0	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v$	$2v+1$	$\cdots$
0	0	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	0	0	$\cdots$
1	1	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	1	1	1	$\cdots$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$v+1$		$\vdots$	$\vdots$	$\vdots$	
$v-1$	$v-1$	$v-1$	$\cdots$	$v-1$	v	$v-1$	$\cdots$	$v-1$	$v-1$	$v-1$	$\cdots$
v	v	v	$\cdots$	v	v	v	$\cdots$	v	v	v	$\cdots$
$v+1$	$v+1$	$v+1$	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$v+1$	$v+1$	$v+1$	$\cdots$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$	
$2v-1$	$2v-1$	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v-1$	$2v-1$	$\cdots$
$2v$	0	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v$	$2v$	$\cdots$
$2v+1$	0	1	$\cdots$	$v-1$	v	$v+1$	$\cdots$	$2v-1$	$2v$	$2v+1$	$\cdots$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$	

The following dominance relation holds for player:

1. $2v \prec^p 2v+1 \prec^p 2v+2 \prec^p \cdots \rightarrow 2v+i \prec^p 2v+i+1$, $i = 0,1,2,\cdots$
2. $v+i-1 \succ^p v-i,\ i = 1,2,\cdots,v$
3. $v+i$, $i = 0,1,2,\cdots,v-1$ is undominated because it is the best response to club's offer $v-1-i$.

The outcome table facing club is as follows:

Table A9. The outcome table of club after first round of elimination when the parties are extremely risk-averse

Club \ Player	v	$v+1$	$\cdots$	$2v-2$	$2v-1$
0	v	$v+1$	$\cdots$	$2v-2$	$2v-1$
1	v	$v+1$	$\cdots$	$2v-2$	$2v-1$
2	v	$v+1$	$\cdots$	$2v-2$	2
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$v-1$	v	$v+1$	$\cdots$	$v-1$	$v-1$
v	v	v	$\cdots$	v	v

The following dominance relation holds for club:

1. $0,1 \prec^c v$
2. $i,\, i=2,\cdots,v$ is undominated because the best response to player's offers $2v+1-i$.

The outcome table facing player is as follows:

Table A10. The outcome table of player after first round of elimination when the parties are extremely risk-averse

Club \ Player	v	$v+1$	$\cdots$	$2v-2$	$2v-1$
0	v	$v+1$	$\cdots$	$2v-2$	$2v-1$
1	v	$v+1$	$\cdots$	$2v-2$	1
2	v	$v+1$	$\cdots$	2	2
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$v-1$	v	$v-1$	$\cdots$	$v-1$	$v-1$
v	v	v	$\cdots$	v	v

The following dominance relation holds for player:

1. $v + j, j = 0, \cdots, v - 1$ is undominated because it is the best response to club's offer $v - 1 - j$.

After the second round ($n = 2$) of elimination, the outcome table for the club is as follows:

Table A11. The outcome table of club after second round of elimination when the parties are extremely risk-averse

Player

Club

	v	$v+1$	$\cdots$	$2v-2$	$2v-1$
2	v	$v+1$	$\cdots$	$2v-2$	2
3	v	$v+1$	$\cdots$	3	3
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$v-1$	v	$v+1$	$\cdots$	$v-1$	$v-1$
v	v	v	$\cdots$	v	v

The following dominance relation holds for club:

1. i, $i = 2,3, \cdots, v$ is undominated because it is the best response to player's offer $2v + 1 - i$.

The outcome table facing player is as follows:

Table A12. The outcome table of player after second round of elimination when the parties are extremely risk-averse

Player

Club

	v	$v+1$	$\cdots$	$2v-2$	$2v-1$
2	v	$v+1$	$\cdots$	2	2
3	v	$v+1$	$\cdots$	3	3

⋮	⋮	⋮		⋮	⋮
$v-1$	v	$v-1$	⋯	$v-1$	$v-1$
v	v	v	⋯	v	v

The following dominance relation holds for player:

1. $2v-1 \prec^p v$
2. $2v-2 \prec^p v$
3. j , $j = v, \cdots, 2v-3$ where $v \geq 3$ $(v < 2v-3)$ is undominated because it is the best response to club's offer $2v-1-j$.

After $n = 2l-1, l = 1,2, \cdots, k$ round elimination, the outcome table facing club is as follows:

Table A13. The outcome table of club after $n = 2l-1$ round of elimination when the parties are extremely risk-averse

Club \ Player	v	$v+1$	⋯	$2v-n-1$	$2v-n$
$n-1$	v	$v+1$	⋯	$2v-n-1$	$2v-n$
n	v	$v+1$	⋯	$2v-n-1$	$2v-n$
$n+1$	v	$v+1$	⋯	$2v-n-1$	$n+1$
⋮	⋮	⋮		⋮	⋮
$v-1$	v	$v+1$	⋯	$v-1$	$v-1$
v	v	v	⋯	v	v

The following dominance relation holds for club:

1. $n-1, n \prec^c v$ because $g^c(n-1,y) = g^c(n,y) = y > v = g^c(v,y)$
2. The circled outcomes are located just under the cells, along which $x + y = 2v$ are the smallest in their columns. That is, club's offer i, $i = n+1, \cdots, v$ is the best response to player's offer $2v+1-i$. Thus, club's offer i, $i = n+1, \cdots, v$ are undominated.

The outcome table of player is as follows:

Table A14. The outcome table of player after $n = 2l - 1$ round of elimination when the parties are extremely risk-averse

Club \ Player	v	$v+1$	$\cdots$	$2v-n-1$	$2v-n$
$n-1$	v	$v+1$	$\cdots$	$2v-n-1$	$2v-n$
n	v	$v+1$	$\cdots$	$2v-n-1$	n
$n+1$	v	$v+1$	$\cdots$	$n+1$	$n+1$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$v-1$	v	$v-1$	$\cdots$	$v-1$	$v-1$
v	v	v	$\cdots$	v	v

The following dominance relation holds for player:

1. $v, v+1, \cdots, 2v-n$ are undominated because $v+i, i = 0, \cdots, v-n$ is the best response to club's offer $v-1-i$.

After $n = 2l, l = 1,2, \cdots, k$ round elimination, the outcome table facing club is as follows:

Table A15. The outcome table of club after $n = 2l$ round of elimination when the parties are extremely risk-averse

Club \ Player	v	$v+1$	$\cdots$	$2v-n-1$	$2v-n$	$2v-n+1$
n	v	$v+1$	$\cdots$	$2v-n-1$	$2v-n$	n
$n+1$	v	$v+1$	$\cdots$	$2v-n-1$	$n+1$	$n+1$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$
$v-1$	v	$v+1$	$\cdots$	$v-1$	$v-1$	$v-1$
v	v	v	$\cdots$	v	v	v

Club's offer i, $i = n, n+1, \cdots, v$ is undominated because it is the best response to player's offer $2v + 1 - i$.

The outcome table facing player is as follows:

Table A16. The outcome table of player after $n = 2l$ round of elimination when the parties are extremely risk-averse

Player

Club	v	$v+1$	$\cdots$	$2v-n-1$	$2v-n$	$2v-n+1$
n	v	$v+1$	$\cdots$	$2v-n-1$	n	n
$n+1$	v	$v+1$	$\cdots$	$n+1$	$n+1$	$n+1$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$
$v-1$	v	$v-1$	$\cdots$	$v-1$	$v-1$	$v-1$
v	v	v	$\cdots$	v	v	v

Player's offers $2v - n + 1, 2v - n$ are weakly dominated by offer v because $g^p(x, 2v - n) = g^p(x, 2v - n + 1) = x \leq v = g^p(x, v)$.

After $n = v$ rounds of elimination, the outcome table facing the club and player is as follows:

1. In case of v being an odd number, that is, $v = 2k - 1$, $k \geq 2$, note that $n = v = 2k - 1$, $2v - n = v$, $n - 1 = v - 1$.

Table A17. The outcome table after $n = v$ round of elimination when the parties are extremely risk-averse (the case of v being an odd number)

Player

Club	v
$v-1$	v
v	v

2. In case of v being an even number, that is $v = 2k,\ k \geq 2$, note that $n = v = 2k,\ 2v - n + 1 = v + 1$.

Table A18. The outcome table after $n = v$ round of elimination when the parties are extremely risk-averse (the case of v being an even number)

		Player	
		v	$v+1$
Club	v	v	v

Q.E.D.