The co-movement of sectoral outputs in Korea

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Abstract

The purpose of this paper is to examine the co-movement of sectoral outputs in Korea. We employ Croux, Forni, and Reichlin's method to estimate dynamic correlations in frequency domain among sectoral outputs. We find that the outputs of manufacturing and service co-move strongly in the short-run as well as in the long-run, but they do not co-move with the output of construction. Consistent with the estimated dynamic correlations, we also find that 20-30% of output fluctuations of manufacturing (service) is explained by spillovers from the service (manufacturing) sector. But, there is no spillover effect between construction and manufacturing (service) sectors. These results suggest that the shock propagation mechanism between manufacturing and service does not work with the construction sector.

Keywords: co-movement, sectoral output, dynamic correlation, cohesion, spillover

JEL Classification: C32, E30, E32

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1 Introduction

GDP co-movement has been one of the important stylized facts in macroeconomy. Soyres and Gaillard (2019, 2022), Blonigen, Piger, and Sly (2014) document business cycle co-movement across countries and provide empirical results that trade linkages are economically and statistically associated with the cross-country GDP correlation. Sectoral GDP co-movement is also another important subject. Given the empirical evidence that the co-movement of output between the non-durable sector and the durable sector, Sudo (2012), Lehn and Winberry (2018), and Horstein and Praschnik (1997) provide models that input-output structure of an economy may explain the sectoral co-movement.[1]

It is true that a common aggregate shock is important to explain the co-movement of sectoral outputs. In contrast, Long and Plosser (1983) show that, in the presence of independent productivity shocks across sectors, agents' optimal choices could cause the co-movement of sectoral outputs in multisector models. Long and Plosser (1987) investigate the data of sectoral outputs and find that the explanatory power of a common aggregate shock for sectoral outputs is significant, but not very large for most sectors. Building on Long and Plosser (1983), Engle and Issler (1995) derive a reduced form for sectoral outputs and perform a special trend-cycle decomposition to find a very similar cyclical behavior across sectors.[2]

The GDP co-movement is often measured by pairwise cross-correlation in the literature. Although the cross-correlation coefficient is the simplest and the most widely used measure of co-movement, it is an imperfect measure in empirical studies. In order to examine business cycle co-movement, the cross-correlation should be computed for a certain cycle of time period but it would be different from the time period of actual business

[1] Erceg and Levin (2002) report that the outputs of durable sector and non-durable sector respond in the same direction to a monetary policy shock. Meanwhile, Barsky, House, and Kimball (2007) argue that the co-movement between the two sectors in standard sticky-price models is highly dependent upon the assumptions about the durable sector's price adjustment.

[2] Several papers examine the sectoral co-movement in Korea. Hwang, Min and Shin (2014) employ a multi-sector time-varying parameter VAR Model to find that the contribution of oil price shock to sectoral co-movement has increased. Kim and Lee (2010) shows that producer service sector enhances the productivity of manufacturing, leading to sectoral co-movement. Yun (2015) also finds that the service sector's linkage with manufacturing has increased since 2000.

cycle in empirical studies. Indeed, the cross-correlation is dependent upon the assumed cycle of time period: 5-year cross-correlation would be different from 10-year cross-correlation.

In this respect, Croux, Forni, and Reichlin (2001) propose a new measure of co-movement which they call dynamic correlation for bivariate series and cohesion for multiple time series. These measures are defined in the frequency domain and thus can be a good complement measure to cross-correlation coefficient in time domain. One advantage of the cohesion is that it can measure co-movement for different cycles from short-run to long-run.

The purpose of this paper is to examine the co-movement of sectoral outputs in Korea. We employ Croux, Forni, and Reichlin's methodology to estimate dynamic correlations among outputs of manufacturing, construction, and service sectors. This exercise enables us to examine which sectors' outputs are synchronized more strongly at different cycles. We also examine the sub-sector output co-movement within manufacturing and within service. Then, we consider a different approach of Diebold and Yilmaz in which we estimate spillover effects across sectors from a VAR model. We find that the spillover effects in time domain are consistent with the dynamic correlations in frequency domain.

The rest of the paper is organized as follows. Section II describes the data and summarizes the methodology proposed by Croux, Forni, and Reichlin. Section III presents the estimation results on the dynamic correlation between sectoral outputs in manufacturing, construction, and service. Section III also provides the estimated cohesion for sub-sector outputs within manufacturing and within service. Section IV provides the estimation results on the spillover effects of sectoral outputs. Section V concludes.

2 Data and Methodology

2.1 Data

We use log differences of the seasonally adjusted real GDP of manufacturing, construction, and services.[3] The sectoral GDP are from the

[3] We exclude agriculture sector which is influenced by weather conditions. We also exclude electricity, gas, and water sector because this sector is under government regulations.

National Income and Product Account for the sample period from 1990 1/4 to 2023 2/4. The summary statistics are presented in Table 1.

Table 1. Summary statistics

	Manufacturing	Construction	Service
Mean	1.43	0.41	1.19
Median	1.49	0.48	1.17
Standard deviation	2.60	2.67	1.08
Autocorrelation	0.23	0.19	0.39
Cross correlation			
Construction	0.11		
Service	0.59	0.15	

Manufacturing is the most rapid growing sector followed by service sector during the sample period. The average growth rate of manufacturing is 1.43% which is higher than 1.19% for service and 0.41% for construction. The volatility of the growth rate is higher in construction and manufacturing than service. The output growth of service is more serially correlated relative to the other two sectors. The cross-correlation coefficient which is the conventional measure of co-movement shows that outputs of manufacturing and service are strongly correlated while the output of construction is less associated with outputs of the other two sectors.

We also consider sub-sectoral GDP in manufacturing and service. The National Income and Product Account classifies manufacturing into 13 sub-sectors as described in Table 2. Service sector is also classified into 11 sub-sectors in Table 2.

Table 2. Sub-sectors in Manufacturing and Service

Manufacturing	
M1	Food, beverages products
M2	Textile and leather products
M3	Wood and paper products, printing and reproduction of recorded media
M4	Coke and refined petroleum products
M5	Chemicals and chemical products
M6	Non-metallic mineral products
M7	Basic metals
M8	Fabricated metal products

M9	Computer, electronic and optical products
M10	Electrical equipment
M11	Machinery equipment
M12	Transportation equipment
M13	Other manufacturing, repair and installation of machinery and equipment
Services	
S1	Wholesale and retail trade
S2	Accommodation and food services
S3	Transportation and storage
S4	Finance and insurance
S5	Real estate
S6	Information and communication
S7	Business activities
S8	Public administration, defense, social security
S9	Education
S10	Human health and social work
S11	Cultural and other services

2.2 Methodology

We briefly explain the methodology proposed by Croux, Forni, and Reichlin to estimate co-movement in the frequency domain. Consider a bivariate stationary time series, $z_t = \{x_t, y_t\}'$, for $t = 1,2,\ldots,T$. Define auto-covariance functions of x_t and y_t as $R_x = E(x_t - \mu_x)(x_{t-j} - \mu_x)$ and $R_y = E(y_t - \mu_y)(y_{t-j} - \mu_y)$, with $\mu_x = E(x_t), \mu_y = E(y_t)$ and cross covariance function as $R_{xy} = E(x_t - \mu_x)(y_{t-j} - \mu_y)$.

Now, we can define the spectral density matrix of z_t, which is Fourier transformation of the variance-covariance functions,

$$f(\lambda) = \begin{pmatrix} f_x(\lambda) & f_{xy}(\lambda) \\ f_{xy}(\lambda) & f_y(\lambda) \end{pmatrix} \text{ for } 0 \leq \lambda \leq 2\pi, \quad (1)$$

and $f_x(\lambda) > 0, f_y(\lambda) > 0$,
where $f_x(\lambda) = \sum_{j=-\infty}^{\infty} R_x(j)\exp(-ij\lambda) = R_x(0) + 2\sum_{j=1}^{\infty} R_x(j)\cos(j\lambda)$,
$f_y(\lambda) = \sum_{j=-\infty}^{\infty} R_y(j)\exp(-ij\lambda) = R_y(0) + \sum_{j=1}^{\infty} R_y(j)\cos(j\lambda)$,
and $f_{xy}(\lambda) = \sum_{j=-\infty}^{\infty} R_{xy}(j)\exp(-ij\lambda)$,

Note that the off-diagonal term represents the cross spectral density which is, in general, complex-valued. CFR replaces the cross spectral density with the co-spectrum, which is equal to the real part of cross spectral density. Define the co-spectrum as $f_{xy}^{*}(\lambda) = \sum_{j=-\infty}^{\infty} R_{xy}(j)\cos(j\lambda)$. Then, dynamic correlation of CFR is

$$\rho_{xy}(\lambda) = \frac{f_{xy}^{*}(\lambda)}{\sqrt{f_x(\lambda) f_y(\lambda)}}. \tag{2}$$

Dynamic correlation measures pairwise co-movement between two variables at different frequencies. Since the frequency λ is inversely related with the period which is a duration of one cycle, low (high) frequency corresponds to long-run (short-run) co-movement between two variables.

To estimate dynamic correlation $\hat{\rho}_{xy}$, kernel-based nonparametric estimators for auto and co-spectral densities are considered as:

$$\hat{f}_{xy}(\lambda) = \sum_{j=1-T}^{T-1} k(j/M)\hat{R}_{xy}(j)\cos(j\lambda), \tag{3}$$

$$\hat{f}_{x}(\lambda) = \hat{R}_{x}(0) + 2\sum_{j=1}^{T-1} k(j/M)\hat{R}_{x}(j)\cos(j\lambda),$$

$$\hat{f}_{y}(\lambda) = \hat{R}_{y}(0) + 2\sum_{j=1}^{T-1} k(j/M)\hat{R}_{y}(j)\cos(j\lambda),$$

where $k(\mathrm{j}/\mathrm{M})$ is a kernel weighting function and M is the bandwidth.[4]

CFR also construct a composite co-movement index called cohesion index, which is a weighted sum of dynamic correlations.

$$\mathrm{COH}(\lambda) = \frac{\sum_{i=1}^{N}\sum_{j\neq i}^{N} w_i w_j \hat{\rho}_{ij}(\lambda)}{\sum_{i=1}^{N}\sum_{j\neq i}^{N} w_i w_j}, \tag{4}$$

where $0 \leq \lambda \leq 2\pi$, and w_i is the weight for the variable i.

[4] We use Bartlett kernel in empirical analysis.

3 Empirical Results

3.1 Dynamic Correlation Between Sectoral Outputs

We examine to what extent the economic activities of different sectors co-move by estimating dynamic correlations in frequency domain. We consider the output growth rates of manufacturing, construction, and service sectors. Figure 1 exhibits the estimated dynamic correlations between a pair of three sectoral output growth along with the 95 percent confidence intervals. The horizontal axis represents the frequency, λ, which is equal to $2\pi j/T$. This relationship between the frequency and the period implies that, for example, λ=0.33 approximately corresponds to a 5-year period of cycle.

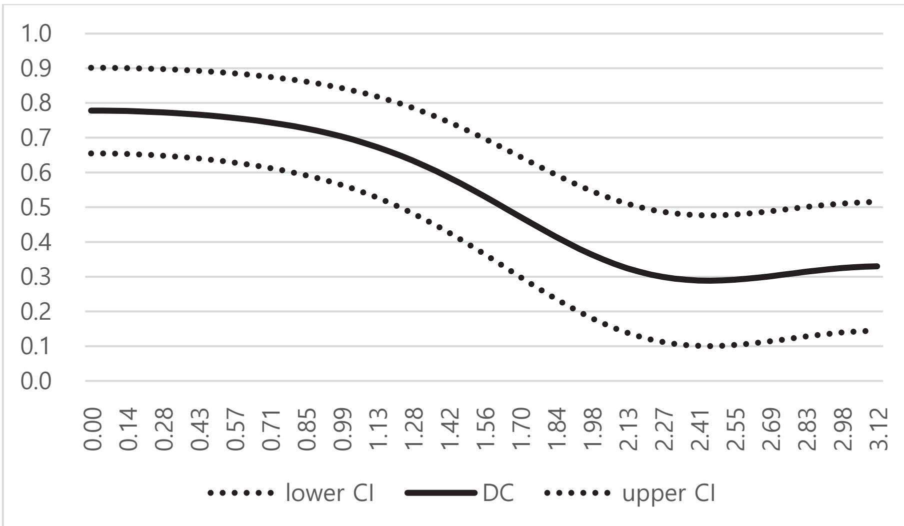
Panel A of Figure 1 illustrates the dynamic correlation between manufacturing and service. The dynamic correlation is statistically significant at all frequencies and it varies depending on the frequency, λ. It is around 0.3 when λ is greater than 2.3, which corresponds to the period of 2.7 quarters. For λ smaller than 2.3, the dynamic correlation increases as λ becomes smaller and it converges to 0.78 as λ approaches to zero.

Since small λ corresponds to long-run relationship, the estimated dynamic correlation suggests that there is relatively weak co-movement in the short-run but very strong co-movement in the long-run between the growth rates of manufacturing sector and service sector.

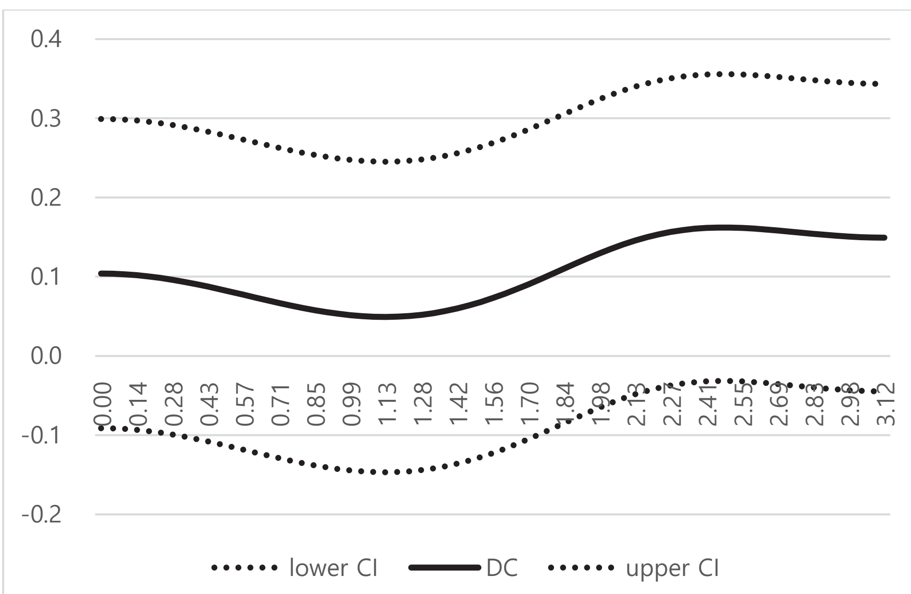
The different correlation between short-run and long-run can be explained by different adjustment cost, including flexibility of price adjustment, between the two sectors. When an economy is hit by an aggregate shock, different cost of adjustment leads to different speed of responses of output in the sectors, which results in weak short-run co-movement and strong long-run co-movement. Another possible explanation is a possibly non-linear propagation mechanism. If a sectoral shock affects the sector immediately and the impacts spill-over into other sectors gradually, the output correlation between the sectors is likely to increase in the long-run.

Figure 1. Dynamic correlations among sectoral output growth

Panel A: Manufacturing and service



Panel B: Manufacturing and construction



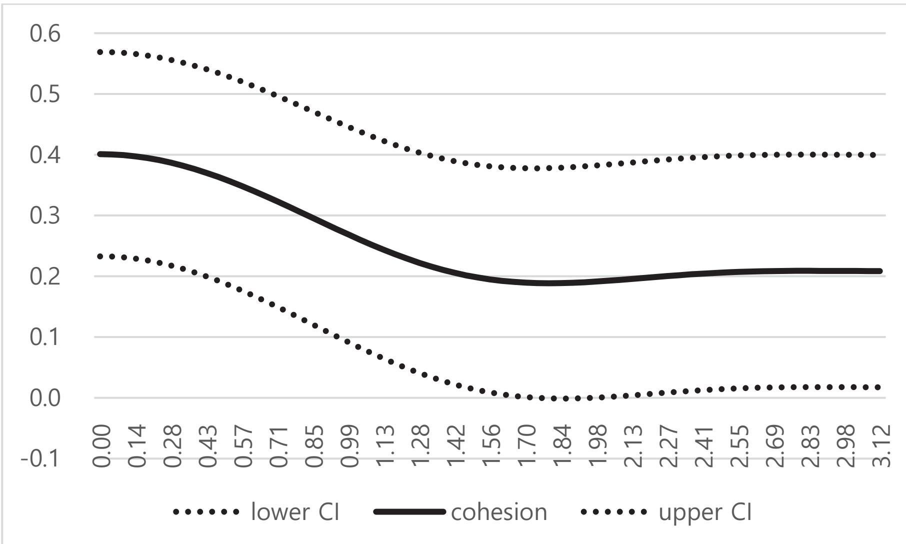
Panel C: Construction and service

Panel B of Figure 1 shows the dynamic correlation between the output growth in manufacturing and construction sectors. It is evident that the dynamic correlation is not economically and statistically significant at all frequencies, suggesting that outputs in the two sectors fluctuate independently. Panel C of Figure 1 shows that the dynamic correlation has a u-shape between the output growth in construction sector and service sector. But it is statistically significant only when λ is smaller than 0.6. This result implies that output of construction is associated with the output of service only in the long-run.

What does this empirical finding suggest? It would be possible that construction sector may not respond to common aggregate shocks. Another possibility is that sectoral shocks in manufacturing or service sectors do not propagate to construction sector or vice versa. The different input-output structure in construction sector is also consistent with the empirical finding. Construction sector uses different intermediated inputs and thus has different value chain, leading to a lack of interconnectedness of construction sector with other sectors.

Figure 2 shows the cohesion between the three sectors which is the average of the three dynamic correlations in Figure 2. The cohesion is around 0.2 when λ is greater than 1.5 while it is increasing as λ is getting smaller.

Figure 2. Cohesion of sectoral outputs



3.2 Co-movement of sub-sectors in Manufacturing and Service

Now we investigate the output co-movements among the sub-sectors within manufacturing sector and within service sector. To this end, we estimate pairwise dynamic correlations in frequency domain between the growth rates of output in manufacturing sub-sectors. Because we have thirteen sub-sectors in manufacturing sector from the National Income and Product Account, seventy-eight pairwise dynamic correlations are estimated. Then, the cohesion within manufacturing is constructed by averaging the seventy-eight dynamic correlations. Similarly, we also construct the cohesion within service. The National Income and Product Account classify service sector into eleven sub-sectors, we need to estimate fifty-five pairwise dynamic correlations. The cohesion within service sector is also a simple average of the estimated dynamic correlations between sub-sectors in service sector.

Figure 3 and Figure 4 show the cohesion within manufacturing sector and the cohesion within service sector. Two interesting findings emerge in Figure 3 and Figure 4. First, both the cohesions within manufacturing and within service are higher in the low frequency than in the high frequency. The cohesion within manufacturing at λ=1.56 (approximately corresponds to 1 year cycle) is 0.27 while it is 0.41 at λ=0. The cohesion within service is also low around 0.13 at λ=1.56 and statistically insignificant but it is much higher and statistically significant in the long-run. It implies that sub-sectoral outputs tend to co-move more closely in the long-run.

Second, the cohesion within manufacturing is higher than the cohesion within service at all frequencies, suggesting that the sub-sectoral outputs in the manufacturing are more likely to be synchronized relative to the sub-sectoral outputs in service sector. In particular, the cohesion within manufacturing is much higher in the high frequency. The cohesion within service in the high frequency, in contrast, is not statistically significant, implying that there is no co-movement in the short-run. But, the two cohesions converge to around 0.4 as λ goes to zero in the long-run.

Figure 3. Cohesion index within Manufacturing sector

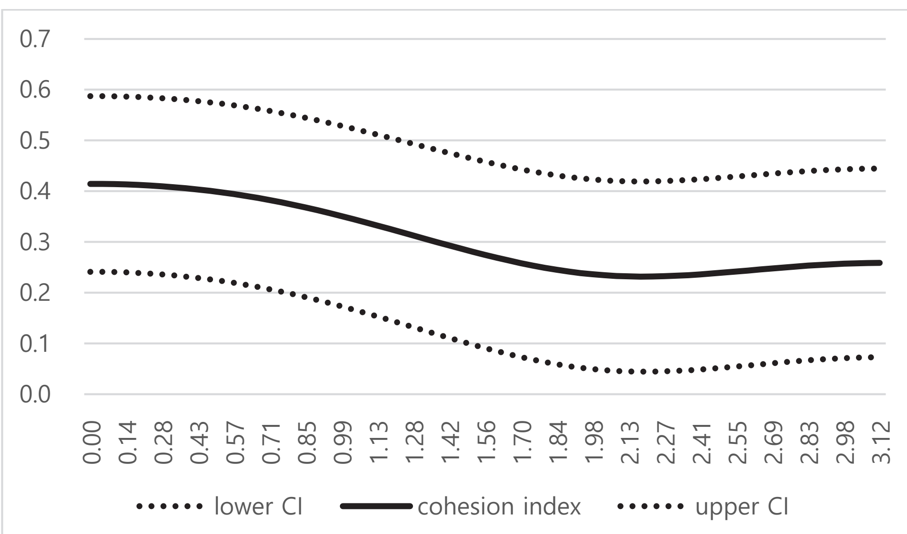
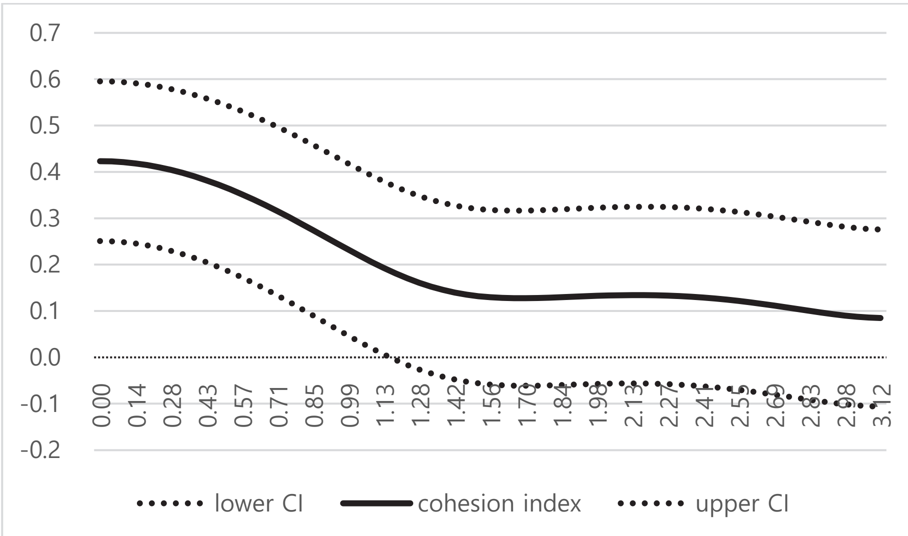


Figure 4. Cohesion index within service sector



3.3 Dynamic Correlations between sub-sector outputs

The cohesion within manufacturing is the average of the dynamic correlations among the sub-sectors. The extent of co-movement of the individual sub-sectors in manufacturing, however, would differ. To examine which sub-sector in manufacturing contributes more to the overall co-movement within manufacturing, we decompose the cohesion within manufacturing into sub-sectoral cohesions. The cohesion of sub-sector i is the average of the dynamic correlations between output growth in sub-sector i and output growth of other sub-sectors in manufacturing.

Table 3 presents the cohesions of each sub-sector in manufacturing for $\lambda = 1.56$, 0.80, 0.33 and 0. Table 3 shows that most of the cohesions are economically and statistically significant, contributing to a strong co-movement within manufacturing as shown in the cohesion in Figure 3. However, the cohesion for M4 (coke and refined petroleum products) at all frequencies and the cohesion for M9 (computer, electronic and optical products) in the short run are not statistically significant. The output growth of coke and refined petroleum products is likely to be related to external factors such as oil price and thus uncorrelated with other sub-sectoral outputs in manufacturing. The output in computer, electronic and optical products is mostly influenced by semiconductor, which is mainly determined by international cycle of semiconductor.

Table 3. Cohesion of Sub-sector Outputs within Manufacturing

	λ=0	λ=0.33 (5 years)	λ=0.80 (2 years)	λ =1.56 (1 year)
M1	0.368*	0.365*	0.327*	0.225*
M2	0.400*	0.404*	0.403*	0.340*
M3	0.465*	0.457*	0.424*	0.361*
M4	0.080	0.070	0.041	0.032
M5	0.460*	0.453*	0.420*	0.344*
M6	0.491*	0.481*	0.440*	0.362*
M7	0.468*	0.471*	0.467*	0.376*
M8	0.398*	0.379*	0.310*	0.221*
M9	0.265*	0.246*	0.160	-0.009
M10	0.528*	0.513*	0.449*	0.298*
M11	0.550*	0.538*	0.487*	0.345*
M12	0.475*	0.478*	0.468*	0.342*
M13	0.438*	0.446*	0.452*	0.333*

Note: Smaller λ corresponds to longer-run relationship between the sub-sectoral outputs.
* denotes the statistical significance at 5% level.

Table 4 presents the cohesion of eleven sub-sectors in service. Figure 4 in the previous sub-section shows that the overall cohesion within service sector is not statistically significant in the short-run. Consistent with Figure 4, only five sub-sectors, including S2 (Accommodation and food services), S3 (transportation and storage), S7 (business activities), S9 (education), and S11 (cultural and other services), co-move with other service sub-sectors in the short run at λ =1.56. Moreover, despite statistical significance, the cohesions in these five sub-sectors are not high, not exceeding 0.3. At $\lambda = 0.33$, however, the cohesions for all the eleven sub-sectors turn out to be economically and statistically significant, leading to strong output co-movement in service as shown in Figure 4 in the long-run.

Table 4. Cohesion of Sub-sector Outputs within Service

	λ=0	λ=0.33	λ=0.80	λ =1.56
S1	0.448*	0.425*	0.350*	0.185
S2	0.499*	0.476*	0.371*	0.213*
S3	0.420*	0.401*	0.331*	0.245*
S4	0.324*	0.296*	0.185	0.002
S5	0.336*	0.297*	0.108	-0.105
S6	0.441*	0.422*	0.324*	0.146
S7	0.493*	0.476*	0.395*	0.254*
S8	0.411*	0.359*	0.136	-0.111
S9	0.478*	0.458*	0.361*	0.198*
S10	0.233*	0.220*	0.179	0.117
S11	0.570*	0.545*	0.436*	0.282*

Note: Smaller λ corresponds to longer-run relationship between the sub-sectoral outputs.
* denotes the statistical significance at the 5% level.

IV. Spillovers among Sectoral Outputs

Cohesion and dynamic correlation do not provide a directional aspect between sectoral outputs. In this section, we take a different approach developed by Diebold and Yilmaz (2012) which enables us to examine directional spillover effects in time domain.[5] We setup a three variable VAR model to estimate spillover effects among output growth of manufacturing, construction, and service.[6]

Table 5 reports the matrix of spillover effects from a generalized impulse response analysis. The cell of i-th row and j-th column in the north-west 3x3 matrix represents the effects on i-th sector from j-th sector. Therefore, the diagonal elements of the matrix indicate the effect of each sector on its own and the off-diagonal elements indicate the spillover effects. The fourth column is the sum of the off-diagonal elements of each row and thus shows the spillover effect on i-th sector from the other two sectors. Similarly, the fourth row shows the spillover effects of j-th sector to other sectors. The sum

[5] A summary of Diebold and Yilmaz is presented in Appendix.
[6] The lag of the VAR is set to be one.

of fourth column or the fourth row is the total spillover effect in the system as shown in the south-east corner of the Table 5. Dividing this total spillover effect by the number of variables gives the percentage of the fluctuation in sectoral output growth explained by spillover effects, which is given in the parenthesis.

Table 5. Spillover Effects of Sectoral Output Growth

	Manufacturing	Construction	Service	From others
Manufacturing	70.2	0.1	29.7	29.8
Construction	0.5	94.7	4.8	5.3
Service	23.3	0.9	75.8	24.2
To others	23.8	1.0	34.5	59.3 (19.8)

Table 5 reports that the total spillover effect is 19.8 percent, implying that 1/5 of the fluctuations in sectoral outputs are explained by spillover effects. This total spillover effect happens to be close to the short-run cohesion in Figure 2.

More interestingly, we find a strong spillover effect between manufacturing sector and service sector. The spillover effect from manufacturing to service is 23.3%, implying that 23.3% of the fluctuation of the output in service is explained by the spillovers from the output of the manufacturing. Similarly, the spillover effect from service to manufacturing is 29.7%. In contrast, it appears that construction sector is less associated with manufacturing or service as the spillover effects in both directions are small. These findings confirm the empirical results on the dynamic correlations in frequency domain in the previous section.[7]

V. Conclusion

We estimate the cohesions in frequency domain to examine sectoral output co-movement in Korea. We find that the outputs of manufacturing

[7] The findings are not unique to Korea. We examine the sectoral outputs in Germany and Japan to find the qualitatively same results. Manufacturing and service sectors are strongly correlated with each other but only weakly correlated with construction sector. Diebold and Yilmaz analyses also find strong spillover effects between manufacturing and service but little spillovers with construction sector.

and service co-move strongly in the short-run as well as in the long-run, but they do not co-move with the output of construction. This finding suggests that construction output does not respond to common aggregate shocks or sectoral shocks in manufacturing or service do not propagate to construction sector. Consistent with the estimated cohesion, we also find that there exist strong spillover effects between outputs of manufacturing and service in both directions. But, there is no spillover effect from and to construction sector.

The empirical findings in this paper have important policy implications. If the sectoral co-movement results from the propagation mechanism across sectors, successful government policies for service industry growth would lead to growth in manufacturing industry and vice versa. In this regard, it is important for the government to choose proper sectors which have strong spillover effects when conducting growth policy.[8] But, the growth policy for manufacturing or service may not have growth effect on construction. At the same time, this finding implies that government policy to boost construction activity would have little impact on other sectors.

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[8] Among the sub-sectors in manufacturing, machinery equipment and transportation equipment have the highest correlations with the service sector in the short- and the long-run. In service sub-sectors, wholesale and retail trade and transportation and storage services are more strongly correlated with the manufacturing sector.

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Appendix: Diebold and Yilmaz (2012)

To examine how sectoral outputs are connected each other in time domain, we estimate a spillover index proposed by Diebold and Yilmaz(2012). We set up a vector autoregressive model,

$$X_t = \sum_{j=1}^{p} \Phi_j X_{t-j} + e_t,$$

where $X_t = (X_{1t}, X_{2t}, X_{3t})'$ is an 3×1 vector of sectoral output growth and Φ_j is 3×3 coefficient matrix. We consider three sectors which are manufacturing, construction, and service sectors. We assume $\mathrm{E}(e_t) = 0, E(e_t e_t') = \Omega$ for all t.

With the assumption of stationarity, X_t has the infinite moving average representation,

$$X_t = \sum_{j=1}^{p} \mathrm{A}_j e_{t-j},$$

where $A_j = \Phi_1 A_{j-1} + \Phi_2 A_{j-2} + \cdots + \Phi_p A_{j-p}$ and $A_0 = I_m$.

We employ generalized forecast error variance decomposition which is invariant to variable ordering. Pesaran and Shin (1998) propose the generalized impulse response function by

$$GI(h) = \Omega_{jj}^{-1/2} A_h \Omega \upsilon_j,$$

where υ_j is an 3×1 selection vector with unity as its *j*-th element and zeros elsewhere.

The h-step ahead forecast error variance of X_i accounted for by X_j can be written as

$$\theta_{ij}^{0}(h) = \frac{\Omega_{ii}^{-1} \sum_{s=0}^{h} (\upsilon_i' A_s \upsilon_j)^2}{\sum_{s=0}^{h} \upsilon_i' A_s \Omega A_s' \upsilon_j}, \ i, j = 1,2,3.$$

Since $\sum_{j=1}^{3} \theta_{ij}^{0}(h) \neq 1$, each entry of $\theta^0(h) = [\theta_{ij}^{0}(h)]$ is normalized by the row sum as

$\theta_{ij}(h) = \theta_{ij}^0 / \sum_{j=1}^{3} \theta_{ij}^0$.

Now, since $\sum_{j=1}^{3} \theta_{ij}(h) = 1$ and $\sum_{i,j=1}^{3} \theta_{ij}(h) = 3$, each row sum of $\theta(h)$ is one and the total sum of the elements in $\theta(h)$ equals the number of the sectors.

As in Diebold and Yilmaz, the total spillover index is defined by as the ratio of the sum of off-diagonal elements to the sum of all the elements in $\theta(h)$. Total Spillover Index $= \frac{\sum_{i,j=1,i\neq j}^{3} \theta_{ij}}{\sum_{i,j=1}^{3} \theta_{ij}} \times 100$.

Following Diebold and Yilmaz, directional spillovers can be defined: The spillover to X_i from other two sectoral output X_js and the spillover from X_i to other two sectoral output X_js are written as

$$\mathrm{SI}(i,.) = \frac{\sum_{j=1,j\neq i}^{3} \theta_{ij}}{\sum_{i,j=1}^{3} \theta_{ij}} = \frac{\sum_{j=1,j\neq i}^{3} \theta_{ij}}{3}.$$

$$\mathrm{SI}(.,j) = \frac{\sum_{i=1,i\neq j}^{3} \theta_{ij}}{\sum_{i,j=1}^{3} \theta_{ij}} = \frac{\sum_{i=1,i\neq j}^{3} \theta_{ij}}{3}.$$

The net spillover effect of X_i is the difference of the spillovers to X_js and the spillovers from X_js,.

Net spillover effect = $\mathrm{SI}(.,i) - \mathrm{SI}(i,.)$